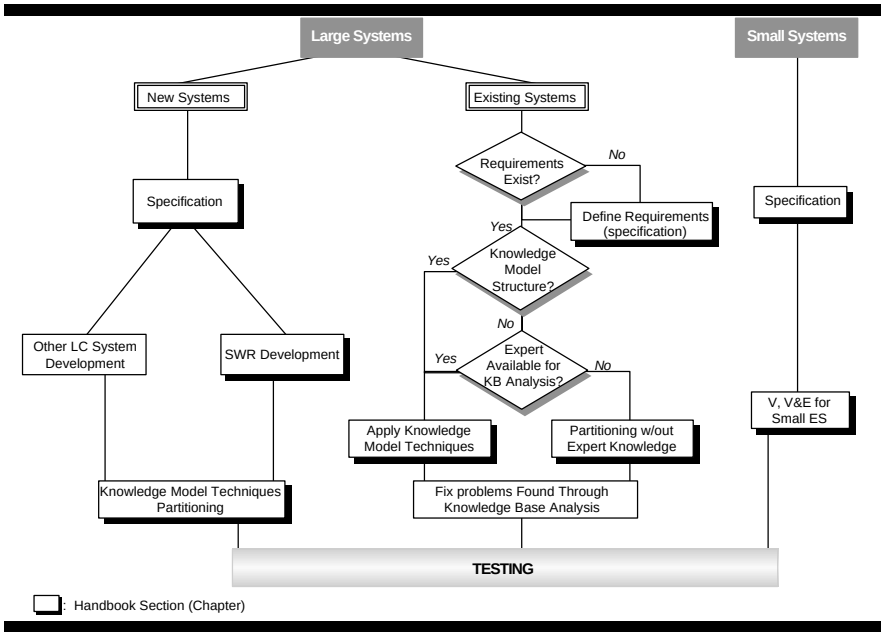
# Verification, Validation, and Evaluation of Expert Systems

*Volume I*

*An FHWA Handbook*

1st Edition (Ver. 1.1)





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16. Abstract

The importance and difficulty of performing verification, validation and evaluation (VV&E) on expert systems cannot be overstated. This is one of the major factors slowing the development and acceptance of expert systems in the transportation community. There is little agreement among experts on how to accomplish the VV&E of expert systems. The complexity and uncertainty related to these tasks has lead to the situation where most expert systems are not adequately tested. In some cases testing is ignored until late in the development cycle, with predictable negative results.

This guide discusses how VV&E should be incorporated into the expert system lifecycle, shows how to partition knowledge bases with and without expert domain knowledge, presents knowledge models, presents methods for validating the underlying experts' knowledge, and presents management issues related to expert systems development and testing. Mathematical proofs for partitioning, consistency, and completeness and visualization of concepts are presented.

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# Dedication

*In memory of Roger E. Hoffman, our deceased colleague whose decency, honor, humor and intellect touched all who worked with him. The Hoffman Regions described in this handbook, a crucial concept in defining the completeness and consistency of expert systems, were conceived by Roger.*

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# 1. Introduction

Roadway engineering and construction pre-date Roman times. Over the centuries, standards in design and construction and the documentation of practice have been raised to very high levels. In the process of modernizing and improving design, construction, and maintenance, new approaches and technologies have been incorporated into civil engineering practice. Initially, many of the new technologies did not achieve the levels of reliability and standardization required by the civil engineering profession. Regrettably, many expert systems fall into this category, due partly to the lack of verification, validation, and evaluation standards.

The goals of expert systems are usually more ambitious than those of conventional or algorithmic programs. They frequently perform not only as problem solvers but also as intelligent assistants and training aids. Expert systems have great potential for capturing the knowledge and experience of current senior professionals and making the expert's wisdom available to others in the form of training aids or technical support tools. Applications include design, operations, inspection, maintenance, training, and many others.

In traditional software engineering, testing [verification, validation and evaluation (VV&E)] is claimed to be an integral part of the design and development process. However, in the field of expert systems, there is little consensus on what testing is necessary or how to perform it. Further, many of the procedures that have been developed are so poorly documented that it is difficult, if not impossible, for them to be reproduced by anyone other than the originator. Also, many procedures used for VV&E were designed to be specific to the particular domain in which they were introduced. The complexity and uncertainty related to these tasks has led to a situation where most expert systems are not adequately tested.

Impelled by the existing environment of lack of consensus among experts and inadequate procedures and tools, the FHWA developed this guideline for expert system verification, validation, and evaluation, complete with software to implement recommended techniques. The guideline is needed because knowledge engineers today do not often design and carry out rigorous test plans for expert systems. The software is necessary because real-world knowledge bases containing hundreds of rules and dozens of variables are difficult for humans to assimilate and evaluate. Computerized verification and validation (V&V) tools would also enable the knowledge engineer to use interim V&V reports to guide knowledge acquisition and coding, something that is too labor-intensive with hand methods. The techniques presented represent a workable solution to a difficult problem. Hopefully they also provide a basis for further enhancements and improvements.

## Basic Definitions

This guide covers *verification, validation, and evaluation* of *expert systems.* An expert system is a computer program that includes a representation of the experience, knowledge, and reasoning processes of an expert. Figure 5.1 shows a six rule expert system that will be used as an example throughout this guide.

*Verification* of an expert system, or any computer system for that matter, is the task of determining that the system is built according to its specifications. *Validation* is the process of determining that the system actually fulfills the purpose for which it was intended. *Evaluation* reflects the acceptance of the system by the end users and its performance in the field. In other words *(Miskell et al, 1989)*:

- **Verify** to show **the system is built right.**
- **Validate** to show **the right system was built.**
- **Evaluate** to show **the usefulness of the system**.

## *Verification*

As stated above, verification asks the question "is the system built right?," i.e., verification is checking that the knowledge base is complete and that the inference engine can properly manipulate this information. Issues raised during verification include:

- Does the design reflect the requirements? Are all of the issues contained in the requirements addressed in the design?
- Does the detailed design reflect the design goals?
- Does the code accurately reflect the detailed design?
- Is the code correct with respect to the language syntax?

When the program has been verified, it is assured that there are no "bugs" or technical errors.

## *Validation*

Validation answers the question "is it the right system?" “is the knowledge base correct?" or "is the program doing the job it was intended to do?" Thus, validation is the determination that the completed expert system performs the functions in the requirements specification and is usable for the intended purposes. It is impossible to have an absolute guarantee that a program satisfies its specification, only a degree of confidence that a program is valid can be obtained. Issues addressed during validation of an expert system include:

- How well do inferences made compare with knowledge and heuristics of experts in the field?
- How well do inferences made compare with historic (known) data?
- What fraction of pertinent empirical observations can be simulated by the system?
- What fraction of model predictions are empirically correct?
- What fraction of the system parameters does the model attempt to mimic?

### *Evaluation*

Evaluation addresses the issue "is the system valuable?" This is reflected by the acceptance of the system by its end users and the performance of the system in its application. Pertinent issues in evaluation are:

- Is the system user friendly, and do the users accept the system?
- Does the expert system offer an improvement over the practices it is intended to supplement?
- Is the system useful as a training tool?
- Is the system maintainable by other than the developers?

To illustrate the difference, the task might be to build a system that computes the serviceability coefficient of pavement. The specifications for the system are contained in textbooks that define the coefficient. To validate the system one must test the serviceability of the program on examples in the texts and other test cases and compare the results of the program with independently computed coefficients for the same examples. It is important to use a test set that covers all the important cases and contains enough examples to make sure that correct results are not just anomalies.

Once the system is validated, the next step is to verify it. This involves completeness and consistency checks and examining for technical correctness using techniques such as are described in this handbook. The final step is evaluation. For the serviceability program, this means giving the system to engineers to use in computing the coefficient. Although the system is known to produce the correct result, it could fail the evaluation because it is too cumbersome to use, requires data that are not readily available, does not really save any effort, does something that can be estimated accurately enough without a computer, solves a problem rarely needed in practice, or produces a result not universally accepted because different people define the coefficient in different ways.

## Need for V&V

It is very important to verify and validate expert systems as well as all other software. When software is part of a machine or structure that can cause death or serious injury, V&V is especially critical. In fact, there have already been failures of expert systems and other software that have resulted in death. For example, a robotized overhead material mover struck an overhead crane at an Alcoa aluminum plant, killing the crane operator, because its narrow-field vision system saw only an interior region of the crane front, a blank field to the robot. In another case, a much-patched system for cancer radiation treatment gave a fatal dose to at least one patient, because the operator overrode the emergency stop; it had given repeated false alarms in past situations.

Expert systems use computational techniques that involve making guesses, just as human experts do. Like human experts, the expert system will be wrong some of the time, even if the expert system contains no errors. The knowledge on which the expert system is based, even if it is the best available, does not completely predict what will happen. For this reason alone, it is important for the human expert to validate that the advice being given by the expert system is sound. This is especially critical when the expert system will be used by persons with less expertise than the expert, who can not themselves judge the accuracy of the advice from the expert system.

In addition to mistakes which an expert system will make because the available knowledge is not sufficient for prediction in every case, expert systems contain only a limited amount of knowledge concentrated in carefully defined knowledge areas. Today's expert systems have no common sense knowledge. They only "know" exactly what has been put into their knowledge bases. There is no underlying truth or fact structure to which it can turn in cases of ambiguity. This means that an expert system containing some errors in its knowledge base can make mistakes that would seem ridiculous to a human, and not realize that a mistake had occurred. [On the other hand, expert systems do not get tired or sick or bored or fall in love, and therefore avoid some of the "careless" mistakes that a person might make, particularly on repetitive problems.] If the expert system does not realize its mistake, and it is being used by a person with limited expertise, there is nobody to detect the error. Therefore, where the expert system is going to be used by someone without expertise, and the decisions made have the potential for harm if made badly, the very best effort at verification and validation is required.

## Problems in Implementing Verification, Validation, and Evaluation for Expert Systems

One of the impediments to a successful V&V effort for expert systems is the nature of expert systems themselves. Expert systems are often employed for working with incomplete or uncertain information or "ill structured" situations (Giarratano and Riley, 1989). These are cases where, as in a diagnostic expert system, not all symptoms for all malfunctions are known in advance. In these situations, reasoning offers the only hope for a good solution. Since expert system specifications often do not provide a precise criterion against which to test, there is a problem in verifying, validating, and evaluating expert systems according to the definitions in section 1. For example, specifying that a speech recognition system should understand speech does not define a testable standard for the system. Some vagueness in the specifications for expert systems is unavoidable; if there are precise enough specifications for a system, it may be more effective to design the system using conventional programming languages.

Another problem in VV&E for expert systems is that expert system languages are not structured to accommodate the relatively unstructured applications. However, rigid structure in implementing the code is a key technique used in writing verifiable code, such as the Cleanroom approach.

Cleanroom software specification (Linger, 1993) begins with a specification of required system behavior and architecture. Many expert systems cannot conform to the rigidity required by this quality control method used principally for conventional programming.

## Intended Audiences for the Handbook

The following table describes the intended audiences for the handbook, and the parts of the handbook that will be most useful to these audiences:

### Table 1-1: Intended Audiences for the Handbook

| **Audience** | **Task to be Performed** | **Part of Handbook** |
|---|---|---|
| Managers | Manage expert system project | Chp. 1: Introduction<br>Chp. 3: Planning And Management |
| Knowledge Engineers | Build new expert systems | Chp. 4: Developing a Verifiable System<br>Chp. 7: Knowledge Modeling<br>Chp. 9: Validating Undelying Knowledge |
| Knowledge Engineers | Perform VV&E on existing systems | Chp. 5: The Basic Proof Method<br>Chp. 6: Finding Partitions without Expert Knowledge<br>Chp. 8: VV&E for Small Systems<br>Chp. 9: Validating Undelying Knowledge |
| Highway Engineers | Ensure that a correct new expert system is built | Chp. 3: VV&E on New Systems<br>Chp. 10: Testing<br>Chp. 11: Evaluation & Manag. Issues |
| Highway Engineers | Ensure that an existing expert system has been validated | Chp. 3: VV&E on Existing Systems<br>Chp. 10: Testing<br>Chp. 11: Evaluation & Manag. Issues |
| Software Researchers | Critique and extend VV&E methods | Chp. 2: V&V: Past Practice<br>Chp. 6: Finding Partitions without Expert Knowledge<br>Chp. 7: Knowledge Modeling<br>Chp. 8: VV&E for Small Systems<br>Chp. 9: Validating Undelying Knowledge |

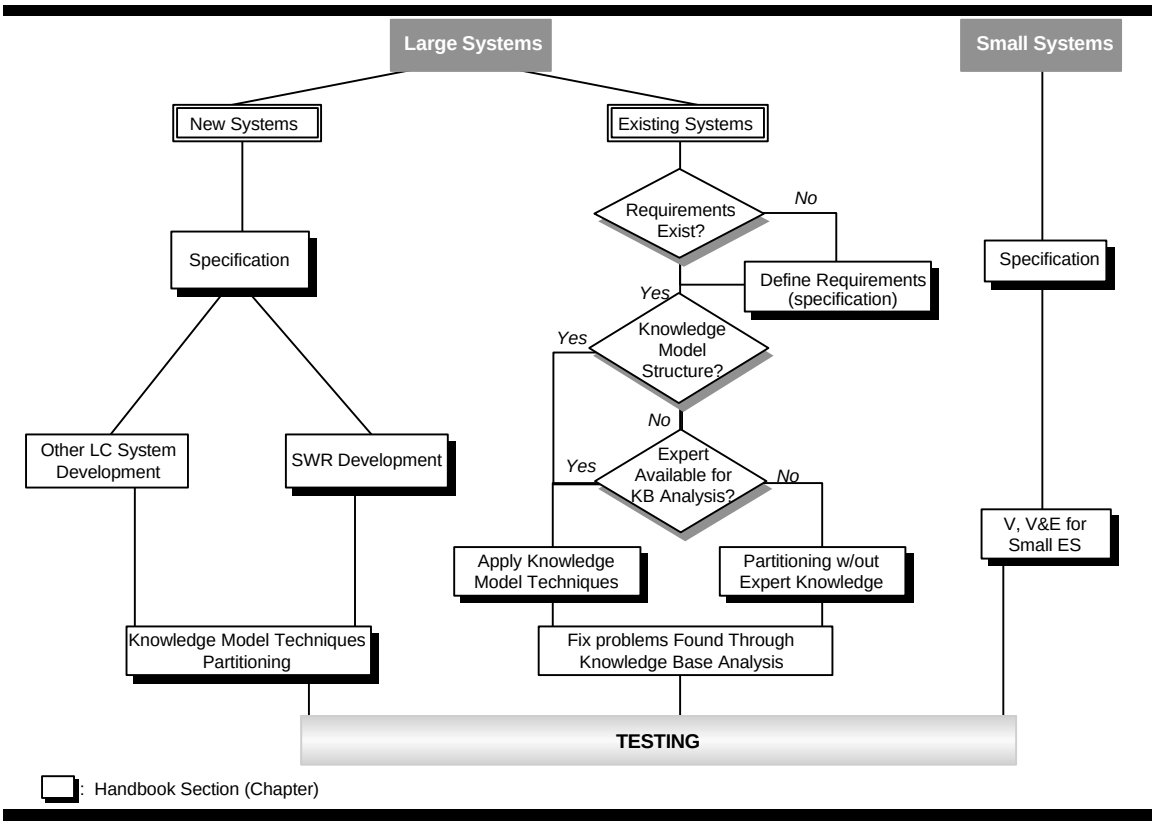


Figure 1.1: The V&V Process

# 2. Planning and Management

The purpose of this chapter is to provide guidance on planning and decision making early in an expert systems project. This concept applies not only to new developments, but to improved thinking and decision making at any stage from development through implementation. This includes planning the verification, validation, and evaluation of an already developed system. The advice given here should aid in developing clear problem definition and thorough system requirements, reflecting realism from both technical and organizational viewpoints. Risk identification information is also provided.

## Introduction

The development, testing and evaluation of an expert system is a demanding process. It is critical in the planning stages that the necessary resources are secured and that the proper development team is assembled. Both the successes and failures in the development of expert systems can usually be traced back to the planning phase of development. The following are important elements of a successful expert system development program:

- Management support in the institution sponsoring the development of the expert system is necessary. This support must include the commitment of both staff and financial resources needed to successfully develop and implement the system.
- The goal of the expert system and the exact uses of the end product must be clearly defined and understood by all involved from executive management to end users. Full knowledge and understanding of the costs and risks involved are also essential.
- Recognized experts in the appropriate technical fields must be available and have sufficient time committed to the expert system development project.
- Influential advocate(s) of the system are essential. Ideally, there should be advocates from both the technical development area and the end user community.
- The end users are pivotal to the development of expert systems and must be involved from the planning through the field evaluation stages. The end users provide definition of the skill level of the user community, information on how problems are addressed in practice versus the prescribed solutions, advice on how the system must function (interact with the user) to be accepted by the user community, and a cadre of supporters to test and promote the system once it is completed.

- Structured planning is recommended for the successful development of a system. This should include the problem/need to be addressed and the system benefits, organizational risk factors, technical risk factors, and user risk factors. Development milestones must be identified and the system demonstrated at each milestone.
- Knowledge elicitation from the experts is vital throughout the duration of the expert system development. It is vital both in terms of building the system and for maintaining interest and continuity throughout the project.
- The verification, validation, and evaluation must be considered in all phases of the system development. Since some aspects of the verification, validation, and evaluation may not be performed by the developers, it is critical that VV&E plans be clearly identified and documented.
- Maintainability must be considered in all phases of the system development. Since the maintenance will probably not be performed by the developers, the system structure must be clear and straightforward. Logical and understandable names should be used for objects and knowledge structures within the system. Clear and complete system documentation is required for effective maintenance.
- The selection of the development tool for an expert system project should be performed by a qualified knowledge engineer or expert systems developer. This is critical because there are significant differences among the development tools. These differences are not explained in available literature and the application should dictate the selection of the development tool with its specific knowledge handling and operational characteristics.

Figure 2-1 shows the initial project planning process. This process can be applied to either a new development of the VV&E for an existing (but not adequately tested) system, or an existing system.

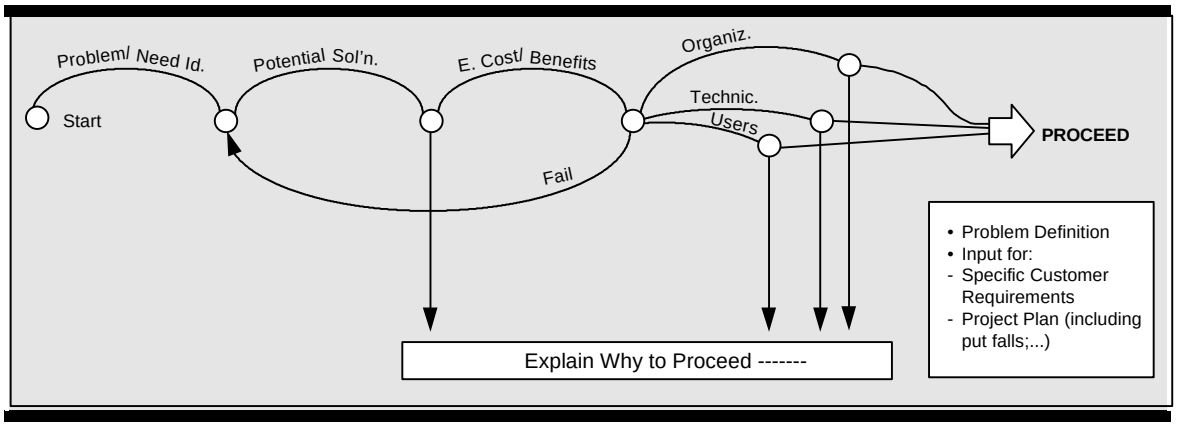


Figure 2-1: Initial Project Planning

## Identify the Need for an Expert System

Before an expert system can be developed, the need has to be established and the problem to be addressed must be clearly identified and defined. It is strongly recommended that this be done in a structured study to include the following issues (Wentworth 1989):

- The problem/need to be addressed and the system benefits.
- Organizational risk factors.
- Technical risk factors.
- User risk factors.

  NOTE: The term risk factors is used in deference to the old adage "if it can go wrong, it will go wrong." The risk factors represent areas where it "will go wrong" if there is any deficiency in planning and common sense.

Once a suitable problem domain has been defined for the expert system, the next task is to narrow the scope of the development effort by clearly defining the set of problems that the system will be expected to solve. The narrower the scope, the better are the chances that the expert system can be successfully built. However, if the scope is too narrow, the application becomes trivial. Judgment must be used in establishing the scope of the system as deterministic methods are not available. In general, it is better to err on the side of too narrow a scope rather than on too broad a scope. If the scope ultimately turns out to be too narrow, it may be relatively easy to broaden the scope by adding more knowledge to the knowledge base. However, if the development tool is too limited, it will be impossible to broaden the scope of the expert system by expanding the knowledge base. This highlights the importance of selecting the proper development tool to fit the particular problem.

Prior to embarking upon an expert system development effort, the expected benefits of such an effort must be clearly defined. There are two categories of benefits that are typically cited as reasons for developing an expert system. One category consists of concrete, quantifiable reasons such as savings of time and money, utility as a training tool, etc. The other category of benefits consists of tangible but not quantifiable reasons. Specifically, the process of developing an expert system will formalize and document the knowledge in a given problem domain, or combine and formalize the expertise from many experts in a given domain. This will result in expanded knowledge and better problem solving techniques in the domain, and provide a mechanism for giving this knowledge wide distribution to the users.

Under the heading of the **problem/need** to be addressed and system benefits, the following should be accomplished:

- The problem or need must be clearly identified and documented.
- The probability of the expert system resolving this problem or need must be described and quantified.
- The application or the output and the use of the output must be clearly defined .
- If standardization of results is desirable, the degree to which the expert system will improve standardization must be estimated.
- The use of the expert system to improve conditions by improving quality of results must be estimated.
- The expected utility of the expert system as a training tool must be described.
- End user involvement for the duration of the development process must be assured.
- Time and money savings based on the projected use of the expert system must be estimated.

Under **organizational risk factors**, suggested requirements and considerations are:

- There must be a dedicated and influential advocate who wants the system to be a success.
- There must be management support for the financial support, staff and time required to build the expert system.
- Management must have realistic expectations regarding the difficulty in developing the expert system.
- Management must have realistic expectations regarding the performance of the developed system.
- The results of the expert system must be applied without excessive management approvals being required.

Once a problem domain has been identified and the initial effort at narrowing the scope of the expert system application completed, the expert(s) whose expertise will be modeled must be selected.

The two main criteria that should be used to identify the expert(s) are:

1. The candidate(s) must be an expert in solving problems in the problem domain of interest and must be recognized as such by the potential user community. The need for the candidate to be an expert in the field is essential for the development of the expert system. The need for the expert to be recognized as such by the potential user community is primarily useful in selling the potential users on the viability of the given system as a useful problem solving tool for them.

2. The expert(s) must be dedicated to the successful development, testing, evaluation, and implementation of the system and be available and willing to spend the time (perhaps months) that will be required to accomplish this. The failure to identify such a person or persons and obtain a firm commitment means that the development project should not be undertaken.

Other useful characteristics for the domain expert(s) to have include the ability to communicate effectively, have an orderly mind, patience and the willingness to teach.

In evaluating **technical risk factors**, the following should be included:

- There must be recognized experts in the field along with general agreement among these experts on the knowledge required to solve the problem the expert system is being developed to address.
- The development team must be identified and arrangements made to assure their dedication to the development and follow-up processes. The availability and personal commitment of all team members must be assured.
- The availability of a manual or automated procedure to be used as a model for the development of the expert system should be considered.
- The required performance of the expert system must be defined (in terms of finding the best solution as compared to senior experts). Unrealistic expectations must be avoided.
- Ambiguity in specifications must be avoided, or if ambiguity does exist, the specifications must be modified to avoid it.
- The scope and range of problems to be addressed by the expert system must be clearly identified.
- Interaction with external programs to run algorithmic routines or for data entry, etc., must be identified.

**User risk factors** must be considered and resolved in the initial planning phases of the expert system development. If representative end users are not involved in the planning and development stages, the system probably will not be accepted by the user community. Issues include:

- The end users must want the system and have a vested interest in its success.
- The computer proficiency and other skills and interests of the end users must be accommodated.
- The environment or conditions under which the system will be operated must be accounted for.

## The Development Team

There are four categories of participants involved in the expert system building process. These are the advocate who champions the building of the expert system, the end users of the expert system, the domain expert(s) whose problem-solving expertise is to be modeled, and the knowledge engineer who actually builds the system. Although in the process of building a given expert system the same person may at various stages of development take on different roles, it is important to recognize that these roles are distinct.

The role of the advocate who champions the development of the expert system is to:

- Identify the need for the system.
- Define the problem domain.
- Identify the intended user community.
- Define the expected benefits that will accrue to the intended audience using the expert system.
- Identify the expert(s) whose expertise will be modeled.
- Choose the knowledge engineer who will develop the system.
- Maintain (or plan for the maintenance of) the finished product.
- Plan and chaperon the entire development process.

The end user is critical in the development of an expert system and must be involved in the entire development process. The end user provides:

- Definition of the skill level of the user community.
- Information on how problems are addressed in the field versus the prescribed solutions.
- Advice on how the system must function (interact with the user) to be accepted by the intended users.
- A cadre of supporters to test and promote the expert system once it is completed.

The domain expert has a dual role in the expert system development process. First, the expert's problem-solving ability serves as the model for the expert system. Second, the expert must assist in quality control on the project and make certain that the expert system faithfully represents a useful portion of the expert's knowledge. In essence, the expert must take some responsibility for ensuring that the expert system faithfully models his expertise. The expert's major task in fulfilling this responsibility is to assist in the design of a comprehensive set of test problems for use in verifying that the expert system actually works.

The knowledge engineer has the task of developing a faithful model of the expert's problem solving ability in the domain of interest. Other tasks which the knowledge engineer must perform are:

- Implement the model of the expert's knowledge.
- Ensure that the implementation is as transparent as possible.
- Document the expert system.
- Test the expert system.

One individual may perform more than one of these functions; however, the end users and their tasks should remain autonomous. If the roles of the domain expert and the knowledge engineer are combined, a second domain expert should review and confirm the technical findings.

## The Test /Evaluation Team

The same four categories of participants involved in expert system verification, validation, and evaluation are involved in the building of the system. However, their roles have changed in some aspects. These are the advocates who champion the building of the expert system, the end users of the expert system, the domain expert(s) whose problem-solving expertise is to be modeled, and the

knowledge engineer who actually builds the system. Although in the process of building a given expert system the same person may at various stages of development take on different roles, it is important to recognize that these roles are distinct.

The role of the advocate who champions the expert system is to:

- Identify the need for system robustness and usefulness.
- Define the problem domain for testing.
- Identify the intended user community.
- Define the expected benefits that will accrue to the testers of the expert system.
- Identify the sites where testing will be conducted.

The end user is critical and must be involved in the entire process from development through implementation. The end user provides:

- Access to a cadre of supporters to test and promote the expert system.
- Information on how problems are addressed in the field versus the prescribed solutions and knowledge on how to "fix" problems on the fly.
- Advice on how the system must function, i.e. interact with the user, to be accepted by the intended users.

The domain expert has a dual role in the expert system development process. First, the expert's problem-solving ability serves as the model for the expert system. Second, the expert must assist in quality control on the project and make certain that the expert system faithfully represents a useful portion of the expert's knowledge. In essence, the expert must take some responsibility for ensuring that the system faithfully models his expertise. The expert's major task in fulfilling this responsibility is to assist in the design of a comprehensive set of test problems for use in verifying that the system actually works.

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- Document the expert system.
- Test the expert system.

One individual may perform more than one of these functions; however, the function of the end users should remain autonomous. If the roles of the domain expert and the knowledge engineer are combined, a second domain expert should review and confirm the technical findings.

## Systems Development Milestones

In developing expert systems a series of development milestones should be used to measure progress and to provide a series of “go/no-go” decision points. These milestones should each represent stages

of development that would provide an improvement in the state-of-the-practice and points where a formal decision by top management to proceed with the development should be made. It is the responsibility of the development advocate to provide criteria for these decisions and to gain management approval of these formal criteria during the planning of the expert systems development.

As an example of this philosophy the following example is provided:

Situation:
A regulatory unit has 2500 paragraphs of regulation to manage. There are about 100 queries per month to these regulations and by mandate responses must be provided in five working days. Files of previous responses are scattered between file cabinets, cardboard boxes and the memory of five remaining experts (all approaching retirement) who know the purpose and the history of the regulatios.

Solutions:
Build an expert system to capture the knowledge of the five remaining experts and to manage the responses to inquiries.

Analysis/Recommendations:
The solution souds wonderful and could even be made to work, but competent and thorough planning and management are required. It should never be assumed that an expert system is the logical answer; an expert system is only a tool and should be evaluated along with other possible approaches. The system shuld be constructed in the following stages:

1. Organize the existing files
2. Develop a system to categorize inquiries
3. Identify typical responses to each category of inquiries
4. Develop a scanning system to automate the reading of inquiries
5. Develp a preliminary response letter based on steps 2,3 and 4
6. Perform VV&E on the developed system

Note that at the end of step 6 a fully developed and tested system will be in place. Also the term "expert system" was not used although in all likelihood an expert system was the abvious tool to use in steps 2,3,4 and 5. Each step represented a clear improvement in the state-of-the-practice and for each step logical "go/no-go" criteria could be prepared during the planning stage of the project.

Figure 2.1.1: System Development Milestones Example

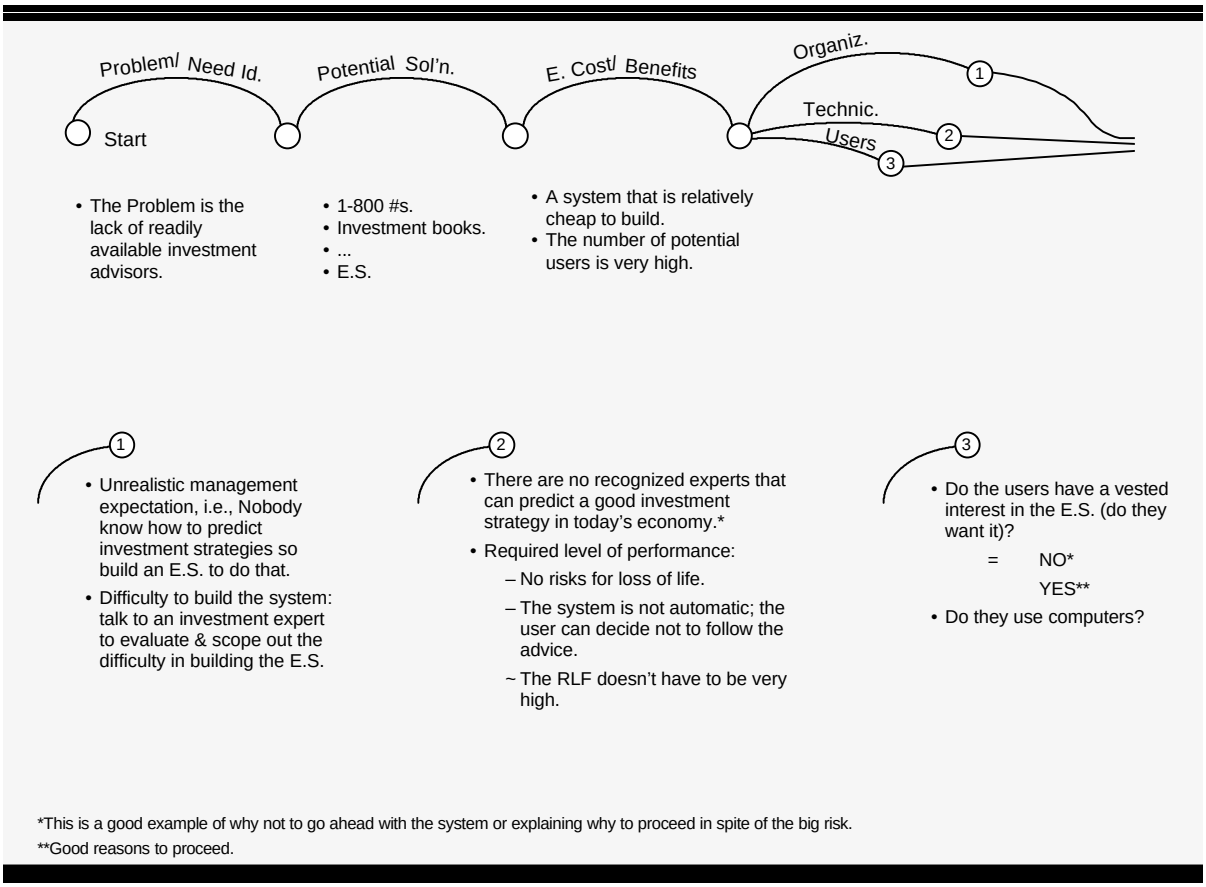



Figure 2.1.2: KB1 Initial Project Planning

# 3. Developing a Verifiable System

This chapter delineates how VV&E should be incorporated into the expert system lifecycle. Although some ideas may be used for revising and/or reengineering existing systems, this chapter is aimed mainly at designing new systems and ensuring that enough VV&E operations are done during the lifecycle so that these systems are verifiable. Included in this process are decisions that should be made during system specification and verification/validation during stepwise development of an expert system.

## Introduction

The proposed lifecycle for the development of expert systems is a compilation of concepts taken from many sources including lifecycle, cleanroom, ect. The compiled system was organized and enhanced based on the experience of its developers to generate a basis for the development of "verifiable" systems. Even though the system allows for some flexibility in the degree of application of each of the system's components, the general outline has to be followed rigorously in order to achieve the objective outlined above.

**The Concept:** Figure 3.1. outlines the general concept for the development of a verifiable system. It includes the following stages:

**Specification:** This step is indispensable in the VV&E process.

**Stepwise Development Process:** This is one of the methods for the development process; other software development methods can be used as long as they include enough structure and verification steps.

Design (1): Start by designing the main parts of the expert system.

Verify (1): Verify that the design complies with the specification.

Implement (1): Implement (code) the first increment.

Verify (1): Verify that the implemented code complies with the design.

Design - Verify - Implement - Verify (2 to n): Loop through the entire process for the 2nd, 3rd, ... nth level until the entire system is complete.

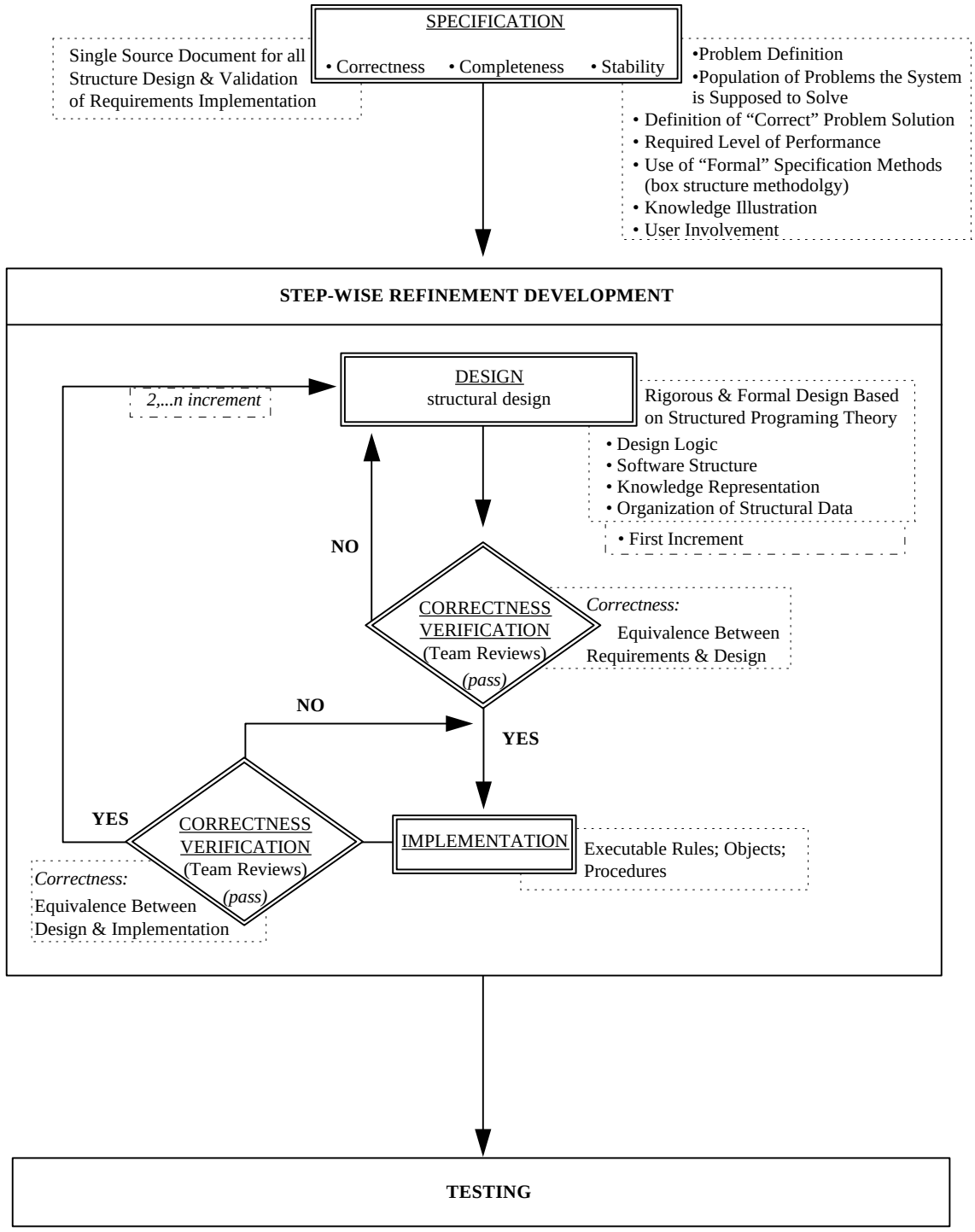



Figure 3.1: Developing a Verifiable System

# Specification

The goal of this stage is to develop the system's specification.

input: software specific customer requirements.

output: software functional and performance requirements.

## *The Importance of Specifications*

Specifications are important for VV&E. As noted in the introduction, *verification* determines if a system meets its specifications; this is meaningless if there are no specifications. *Validation* determines if a system does what is needed; this is only possible if it has been decided what a system is supposed to do. The results of these decisions are *specifications*.

At the specification stage the emphasis is on producing a clear identification of:

- What is to be produced?
- When to produce it?
- What are the resources required?

The issue is to find a trade-off between the requirements specification (client) and the resources (time and money). The use of formal approaches (formal notation i.e., the Structured Analysis [SA; De Marco 1978], the Software Requirements Engineering Methodology [SREM; Alford 1978], the Structured Analysis and Design Technique [SADT; a trademark of SofTech],) proved to be very useful in this process. This is especially important to the V&V task because of the clarification provided by the use of these methods.

Functional Specification (FS): Specification of functions to be performed by the system and the constraints within which it must work.

Acceptance Test Specification: Test definition:

- Who will perform the test(s)?
- When (at what point)?
- How do we insure that the system behaves according to the FS ?
- Include V&V Techniques to be used and when (at which time).

In addition to the above mentioned items, the following items should also be addressed:

- A clear definition of the population of problems the expert system is supposed to solve.
- A provision of test and development samples.
- The required level of performance.
- A clear definition of what constitutes a correct problem solution verification:
    - Is it possible to collect inputs that could possibly solve the problem?
    - Is it possible to compute the proposed output from the input validation?

- Can the experts certify that the specifications, if properly implemented, would solve the evaluation problem?
  - Can experts judge that the system is worth the probable cost?
  - Can experts judge that the system would be useful in practice?
  - Is it possible to build a system that could be integrated with other components as necessary?

## *The General Form of Specifications*

For the formal proof techniques presented in the following chapters, it is useful to have a general representation of a specification. Most specifications are based on the following form:

> For some subset S of the input space of an expert system, and
> for all X in S,
> the output of the system satisfies some proposition P.

## *Defining Specifications*

It is particularly important to define specifications for the critical cases the expert system may encounter. A *critical case* for an expert system is a set or range of input data on which failure of the expert system to perform correctly causes an unacceptable, perhaps catastrophic, failure of the system of which the expert system is a part.

There are several steps in defining and verifying specifications for an expert system:

- Gather informal requirements from experts, with particular attention to defining the critical cases.
- Obtain expert certification of the specifications.
- Validate informal descriptions of the specifications with experts.
- Validate the translation of informal specifications into the formal notation used in the knowledge base.
- Validate the formal statement of the requirements using symbolic evaluation.

Each step is detailed in a section below, with particular attention to critical cases.

## *Gather Informal Descriptions of Specifications*

The first step in verifying specifications is to gather a complete set of requirements. Only the domain expert(s) can provide this list. Ideally, during the original knowledge acquisition phase for the expert system, the knowledge engineer gathered, documented, and validated the critical cases. If the informally stated requirements are not available, however, gathering them is the first necessary step in verifying the correctness of an expert system.

Typically, to gather the critical cases, the knowledge engineer should ask the domain expert(s) to list critical cases, and to keep a careful record of them. As with most knowledge acquisition tasks, it is important to ask for the following information:

- General principles, e.g. "What are the critical performance requirements for this expert system?"
- Specific projects, and the critical performance requirements found in those projects. To get this information, the knowledge engineer should ask the expert(s) to tell him about their projects and experiences that are within the scope of the knowledge base. The purpose of this is that by reviewing the specific projects the expert's memory will be spur. This process will help the engineer to decide what the critical cases really are.

In gathering a set of critical cases, it is important to let the domain experts describe critical cases in their own words and notation, not in the notation of the expert system. This is because the expert system may have missed a critical variable that may be needed to recognize a critical case. If the knowledge engineer asks the expert to verify knowledge base gobbledygook, the expert may become too distracted to think of a critical case not described with the incomplete set of variables used in the incomplete knowledge base.

### *Obtain Expert Certification of the Specifications*

It is important that the knowledge engineer impel the expert(s) to certify the specifications, especially those concerning the critical cases. This is a vital step in the process because the expert system will be built to meet and tested against the specifications. If the specifications are in error, the expert system will almost surely fail to perform properly.

In order to obtain meaningful certification of the specifications, the knowledge engineer must make sure that the expert focuses on a careful review of the specifications. Among the ways to obtain this focus are:

- Have a group of experts reach consensus on the specifications, with the knowledge engineer functioning as a moderator. In this role, the engineer will:
    - Be familiar with the ongoing discussion, and in addition, will be in a position to solicit important issues that must be resolved.
    - Ensure that the experts address those issues and reach an agreement.
- Have the expert(s) sign off on the specifications.

### *Validating Informal Descriptions of Specifications*

For systems where correct performance is critical, the next step in validating specifications of the expert system is to validate the informal descriptions of critical cases. The basic method for validation is that of cultural consensus, described in the chapter, "Validating Expert Knowledge." In this method, experts, ideally those who have not provided the specifications, are used to validate the correctness of those specifications.

There are two questions that should be asked concerning the informal list of critical cases to validate: is the set of critical cases complete, and are the critical cases correct? To validate completeness, the knowledge engineer should conduct interviews with experts who have not contributed to the critical case list. This interview is similar to the one used to gather the list of critical cases, with one additional step: at the end of the interview, ask the expert to certify not just the critical cases the expert proposed, but the entire list of critical cases gathered so far, including those that were added during the

interview. After additional experts no longer provide new critical cases, the entire list gathered has been validated to a confidence level depending on the number of experts who certify the list. Chapter 9, "Validating Underlying Knowledge", discusses these confidence levels in more detail.

### *Validating the Translation of Informal Descriptions*

To validate the critical cases, the informal descriptions must be translated into formal statements in the language of the knowledge base. The goal of this translation is to produce statements of the form:

> if H1 and H2 ... and Hn then C1 and C2...and Cn.

The H's should be stated in terms of input variables of the expert system, and the C's should be possible conclusions of the expert system.

The translation into a knowledge base language is a process that can introduce errors. For example, for Knowledge Base 1 a critical case in the informal language of an expert might be, "If the client doesn't have a lot of money, he/she should first build a savings account." The closest that one can come to expressing this in the language of Knowledge Base 1 is:

> If "Discretionary income exists" = no
>
> then investment = "bank account".

A financial planner would probably consider "Discretionary income exists" an inadequate translation of "the client doesn't have a lot of money"; Knowledge Base 1 does not even ask about existing savings or most other assets.

As this example illustrates, the translation of expert knowledge into the formal knowledge language of an expert system is one of the tasks where errors can creep into the expert system. To have a truly validated expert system, the translation has to be validated. Although this is rarely done, items can be created for validation as follows:

- Is <expert's statement of a critical case>
- equivalent to <the same critical case in the knowledge language>

These items form the basis for a cultural consensus test for a set of knowledge engineers (see chapter 9 "Validating Underlying Knowledge"). When asking knowledge engineers to validate the translation of critical cases, it is important to:

- Use knowledge engineers who have not built the knowledge base.
- Give the validating knowledge engineers the opportunity to familiarize themselves with the knowledge language before examining the individual items.

In translating the informal requirements into formal knowledge base statements, there are some typical kinds of errors, as discussed below:

- False negatives in the input variables: One problem in knowledge translation results from the fact that a symptom is often used in a knowledge base to stand for an underlying condition; in the above example, for example, "no discretionary income" stands for "has no money." However, few observations are 100 percent reliable. If a single symptom is used to test for a condition in a knowledge base, a false negative of that symptom will produce an error in what the expert system does.

The solution to the false negative problem is to separate symptoms and underlying conditions in the knowledge base. If C is a condition, the knowledge base should contain a rule of the form:

if S1 or S2 or ... Sn then C (Rule C).

Where S1 through Sn are a set of symptoms such that the probability of false negatives in all the S's Is less than some agreed-on threshold. Outside of Rule C, and similar condition-inferring rules, the S's should not appear when a condition (i.e., C) is intended. Therefore, every occurrence of an S outside of a condition-inferring rule should be validated by expert(s).

In the case where a single symptom has such low false negatives that it identifies C by itself below the acceptable error threshold, it is unnecessary to separate the symptom and condition in the knowledge base:

- Missing input variables: An expert learns to observe many symptoms of possible problems. An expert system may use only a small number of variables. Whether the small number of variables is adequate is a matter that experts must validate. It is important to ask experts what data they gather in looking at problems covered by the knowledge base. If the expert looks at more than the expert system, for example variable X, then:

  Can the expert get along without <variable X>

  is a knowledge item that should be validated (see chapter 9).

## *Validation of Formalized Requirements*

At this point, the critical cases have been transformed into a set of statements of the form:

if H1 and H2 and ... and Hn then C1 and ... Cm (name: f1).

Formal verification methods for specifications in this form are discussed in the chapters on knowledge modeling and verification techniques for small systems.

Figure 3.2 outlines the steps to be considered at the specification stage and figure 3.2.1 shows their implementation to knowledge base 1.

Other Issues to be addressed at this stage:

- Project Plan:  Breakdown of the work; manpower figures, milestones, ect.
- Quality Management Plan:  Quality Control.

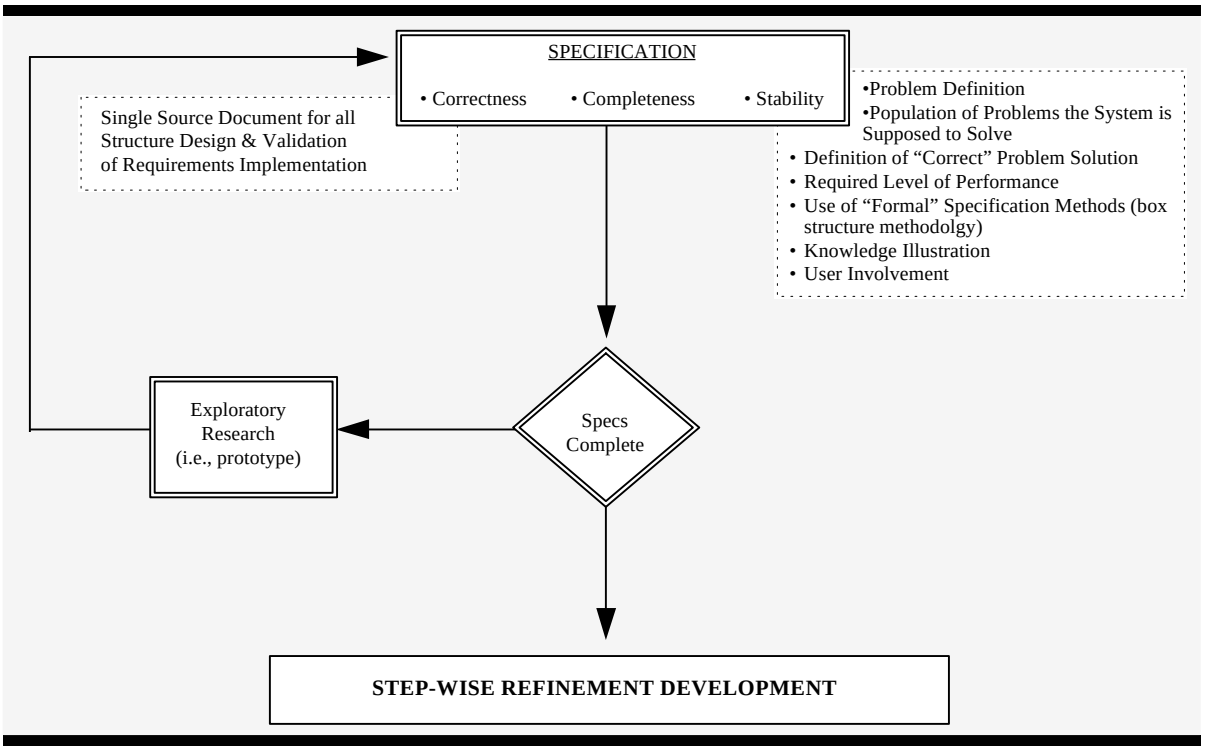



Figure 3.2:  Specification

Problem Definition:

- The lack of readily available investment advice.
- Develop a system that will advise the user on investment strategies.

Population of Problems the System is supposed to solve:

- Investment advice to people with less than $ 1 Million to invest.

Definition of Correct Problem Solution:

- An investment strategy is always suggested.
- Proposed solution should be affordable
- The investor is comfortable with the advise.

Required level of performance:

- As good as 70% of the expert(s) [Define: good 70% of the time or 70% as good all the time].
- The system should always recommend an affordable investment even if it has to be a conservative one.

Note: Knowledge Acquisition & User Involvement:

Figure 3.2.1: KB1 Specification

## Step-Wise Refinement Development

At this stage, a mapping of the system functions (from FS) into software components will occur and the overall System Structure (Architecture) must be defined. The use of the following Box Structure Methodology will help in this process.

### *Software Structure*

The general software architecture should consist of:

- Software Components (for each software component, determine its purpose, functionality, interface, and data requirements).
- Structure & Flow.

### *Box Structure Methodology*

- Black Box: External view of the system. This provides a system description of the user visible system inputs and responses. No details on the internal structure and operations are provided.
- State Machine: Intermediate system view. This decomposes the internal state structure from the BB description of the system.
- Clear Box: Internal view of the system operations on inputs and internal state data.

If the box structure methodology, is to be used, the first level/increment should consist of the overall design taken as deep as possible (using black boxes for functions and sub-systems). At every

subsequent increment the design should be taken deeper, two to three level down, until all the boxes are replaced by their respective functions/subsystems.

## *Design Refinement*

The Top Level Design:

Given the specifications for the system as a whole, a top level software module is designed with the following properties:

- The design for the top level software is written in a language, which may be but is usually not a compilable programming language. Any language which has a precisely enough defined syntax and semantics to unambiguously define what the design does when executed, and to carry out required correctness proofs of the software can be used. Languages tha fit these requirements are called design languages. The process of rendering axiomized software into a design language is called designing the software.

- The software design can be translated from its existing language into a compilable programming language. Techniques for doing this translation for standard knowledge models will be presented later. The software design can be proved correct. In particular, the software can be proved to be complete, consistent and to satisfy its specifications, under the assumption that any other functions or other software modules used within the current object of proof satisfy some written, precise, mathematical specifications.

Refining the Design:

Once the disign process has been started, a modification of the familiar successive refinement lifecycle adds detail to the design. Detail is added in two ways:

- Software modules which have been axiomized but not designed can be designed.

- Software that has been designed can be translated into a language that is closer to, or is, a compilable programming language.

Verifying the Design:

A design is verified when it has been proved that the designed module is complete, consistent and satisfied its specifications.

- A module is complete iff for all points in its input space, some values of the outputs and behaviors required to instantiate the specifications are computed.

- A module is consistent iff it is possible, both mathematically and under the constraints imposed by knowledge in the area of application, for all the output values and behaviors to be true at the same time.

- A module satisfies its specifications iff its specifications are true when instantiated with any input values and any outputs or behaviors produced from those inputs.

Completing the Design:

A design is complete when all software modules appearing in the design have been axiomized, designed and verified.

---

From Specifications:
1. An Investment strategy is always suggested:
List of possible investments strategy for KB1:
- Stocks.
- Saving Accounts.

Do Nothing (Although this is a good choice for many instances, it is not considered for the example).
*Note: The list of output might be incomplete at this stage (i.e., may discover other possible strategies down the line).*
Define the specifications in terms of these newly defined list of output.
2. Proposed solution should be affordable:
- When is stock affordable?
- When is Saving Account affordable?

Interaction with the expert(s)
Depending on the complex nature of the questions to be answered, we may find out that other things might be needed:
- Interaction with data bases.
- Algorithmic routines.
- Sub Expert systems.

For KB1:
The expert determined that stocks are affordable if "Discretionary Income" exists.
We have to define "Discretionary Income" in a measurable manner.
From the interaction with the expert, we introduce the concept that in order to have "DI", the investor has to have:

Some savings (> $ 3000.).
A luxury item (Boat/ Luxury Car).

n.b.: 1. Keep careful records of interaction with the expert(s).
2. One of the products of these steps are expert(s) verifiable statements about the knowledge domain.

*i.e., Stocks are affordable if there is savings and a luxury item.*
These will be used for carrying out formal proof procedures. In a high risks situation (see table 3.1) these statements should be verified by enough experts to get the required level of confidence (see chapter 9).
We have preliminary design information that consists of:
1. An expert sub-system to determine affordability.
2. An expert sub-system for risk tolerance.

---

3. An expert sub-system which makes an investment category decision using 1 & 2.
n.b.: This is very useful for designing a well structured system.
Refer to chapter 7, "Knowledge Modeling", and pick a knowledge model that fits the preliminary design information.

Figure 3.2.2: KB1 Design

## Implementation

In the implementation step, a software module is translated from its current design language into a compilable language. The source code resulting from that translation must be verified, i.e. shown to be complete, consistent and to satisfy its specifications.

Implementation is the last step in a series of design and translation steps that turn an initial high level specification for the system as a whole into compilable code. The stepwise refinement process that produces the code from the initial specification uses the following refinement operations:

- Design of a module that has been specified but not yet designed.
- Specification of a module that is used in a designed module but has not yet been specified.
- Translation of a module from one design language to another language, usually one that is more detailed and closer to a compilable language.

At the Implementation stage, the main objective is the creation of a complete executable system, including software to carry out all processes specified in clear or black boxes, according to constraints on those parts of the system. The system is comprised of executable rules, objects, procedures, etc., that:

- Satisfy requirements of the system as a whole .
- Are executable functions that are equivalent to abstract functions specified in the design.

For example, the design may specify a function that determines that the user is rich. The implementation may check the bank account, kind of car owned, etc. However, it may not catch certain rich people because it does not check art owned. In this case, the implementation fails to carry out the abstract function required of it. In general, the computer bases a conclusion on less observed data than an expert, and simplifies the inference an expert makes to one that is just based on the small set of data the computer looks at.

The implementation stage should consist of the following steps:

1. Determine the high level structure of the system to be implemented.
2. Define communication between subsystems Implementation.
3. Provide a detailed definition of subsystems.

4. Select the implementation tool.

5. Execute the implementation in the tool.

## Constraints on Design and Implementation

The following constraints apply to the operations in the stepwise refinement process:

Specification of a module must include all properties that are used in any existing verifications of other modules.

- No module can be designed before it is specified.
- Designs must be proved to satisfy their specifications.
- Translations must preserve specified properties of the source module (being translated) in the destination module (the result of the translation).

## Correctness Verification

### *Design vs. Specification*

The overall result of this is a proof that any system that satisfies all the design documents is correct (i.e., complete, consistent, stable, satisfies requirements imposed by subject) provided that the parts not yet designed or implemented have properties as required by clear box theorems and the models of knowledge, or specified by the expert.

### *Code vs. Design*

The equivalence between requirements and implementation must be proven. Previous results together with proof of equivalence of design and implementation may be used. This may take the form of a cleanroom-type layered correctness proof in which all boxes are clear and implemented, with the top part constituting the previous proof of the equivalence of requirements and design.

Depending on the complexity of the problem and the consequence of failure, this process is to be accomplished by the developer(s) (Level I), the developer(s) and two members of the organization (Level II), or a separate verification team (level III). Table 3.1 is to be used as a guide in determining the level of the project. figure 4.3 shows the process and figure 3.3.1 is the implementation to knowledge base 1.

Table 3.1: Level of Effort for the Correctness Verification Stage

| | Consequence of Failure | | | | |
|---|---|---|---|---|---|
| **Complexity** | **Loss of Life** | **Injury** | **High $$$** | **Inconvenience** | **Other(lC,...)** |
| **Very Complex** | III | III | III | II | I |
| **Medium** | III | III | II | I | I |
| **Simple** | III | III | II | I | I |

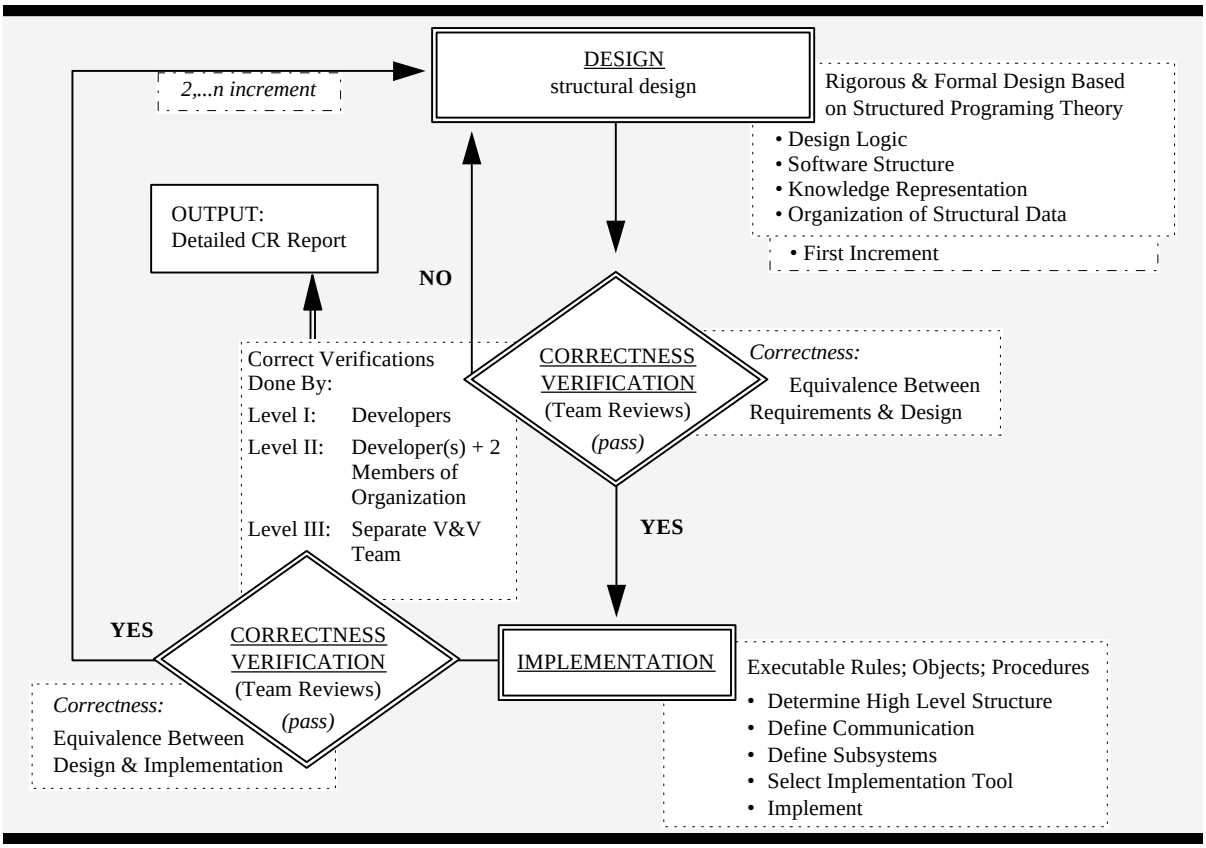



Figure 3.3: Correctness Verification

**Step 1 -- Determine the high level structure of the system to be implemented**: From the design stage, it was determined that the expert system consists of 3 subsystems, discretionary income (DI),

risk tolerance (RT) and type of investment (INV). The structure of the system can be expressed by the function:

Investment = INV( DI( boat, "luxury car", "savings account"),

RT( stocks, "lottery tickets"))

This expresses the fact that the output of DI and RT are inputs to INV.

**Step 2 -- Define communication between subsystems**: The output of DI and RT must be sufficiently fine-grained to distinguish cases where different investments are indicated. Since there are only 2 investments in this example system, only 2 values are required as output for each of these subsystems; use *high* and *low* for risk tolerance, and *yes* and *no* for discretionary income. At this point, the inputs, outputs and communication between subsystems have all been defined.

**Step 3 -- Detailed definition of subsystems**: In this stage, the expert information collected in the design step will be converted into precise logical statements; this process will be illustrated on the DI subsystem.

The condition that must be true to have discretionary income is:

A = (Savings > $3000) (1)

AND ( "Own Boat" = yes OR "Own Car" = yes )

The expert information about discretionary income can be formalized as:

A IMPLIES ("discretionary income" = yes) (2)

NOT A IMPLIES ("discretionary income" = yes) (3)

**Step 4 -- Selection of implementation tool**: At this point, there is enough information to choose a tool in which to implement the expert system. The requirements on the tool are:

- Provide for communication between subsystems.
- Express rules such as (2) and (3).

Most rule-based expert system shells meet these requirements. Although the order of information in the knowledge base must be slightly different in forward and backward chaining implementations, either form of inference engine can be used to implement this knowledge base.

**Step 5 -- Implementation in the tool**: The rule-based-shell implementation will be written in two steps: first as a generic rule-based implementation, finally as an implementation in CLIPS.

**Step 5.1 -- A generic rule-based implementation**: Rule-based shells typically allow menu, fill-in and yes-no questions. The following questions will gather the necessary information for discretionary income:

| QUESTION TEXT | TYPE |
|---|---|
| What is your savings balance? | fill-in |
| Do you own a boat | yes-no |
| Do you own a luxury car | yes-no |

The inputs and outputs can be represented inside the expert system by the following variables:

| VARIABLE | TYPE | VALUE |
|---|---|---|
| savings | numerical | >= 0 |
| "Do you own a boat" | boolean | yes or no |
| "Do you own a luxury car" | boolean | yes or no |
| "discretionary income" | enumerated values | high or low |

Now put the knowledge in statements (2) and (3) into the rule form of rule-based shells. Rule based shells encode information in the following form:

• Rules are of the form:

IF <conditions> then <inferences> and <actions>

• <conditions> are built from simple requirements with the logical operations AND, OR and NOT.

• Many of the simple requirements can be written in the forms such as VARIABLE = VALUE, or more generally:

VARIABLE REL VALUE, where REL is one of the relations

=, >, <, >=. <=

• Inferences can also be written in the form:

VARIABLE = VALUE, i.e. VARIABLE is set equal to VALUE.

Actions are dependent on particular shells, and will be deferred at this time.

Using the above notation, (2) can be written as:

IF (Savings > $3000)  (4)

AND  ( "Do you own a boat" = yes

OR "Do you own a luxury car" = yes )

THEN "Discretionary income" = yes

(3) can be put into rule form as:

If NOT <if part of (4) THEN "Discretionary income" = no  (5)

Alternatively and more usually, a rule implementing (3) is written in a form in which the NOT is applied individually to the simple requirements contained in the "IF" part, rather than to a complicated expression built up from requirements. *DeMorgan's Laws* in mathematical logic:

NOT (A OR B) = NOT A AND NOT B  (6)

NOT (A AND B) = NOT A OR NOT B

Using (6) repeatedly transforms (5) to:

IF (NOT Savings > $3000)  (7)

OR

( NOT "Do you own a boat" = yes

AND NOT "Do you own a luxury car" = yes )

THEN "Discretionary income" = no

Simplifying the simple conditions using the following relations,

(NOT Savings > $3000) = (Savings <= $3000)  (8)

( NOT "Do you own a boat" = yes)

= ("Do you own a boat" = no)

( NOT "Do you own a luxury car" = yes)

= ("Do you own a luxury car" = no)

Substituting (8) into (7) gives:

IF (Savings <= $3000) (9)

OR ( "Do you own a boat" = no

AND "Do you own a luxury car" = no )

THEN "Discretionary income" = no

Figure 5.1 shows an expert system in a generic rule-based shell language that implements the discretionary income, risk tolerance and investment subsystems. The result is a small knowledge base (called Knowledge Base 1) that implements the investment expert system. [Note: Knowledge Base 1 leaves out the savings requirement, to further simplify the example when it is used to illustrate verification and validation.]

**Step 5.2 -- Implementation in CLIPS**

Once a generic knowledge base has been written, it must be translated into the language of a particular shell. Shown below is an implementation of the generic knowledge base in CLIPS. The CLIPS is fairly close to the generic rule-based KB. The main differences are:

**rule syntax**: Rules in CLIPS have the following syntax:

```
(defrule <RULE NAME> <COMMENT>
 <LIST OF CONDITIONS>
 =>
 <LIST OF ACTIONS AND INFERENCES>
)
```

**implementation of the AND operation**: The AND operation can be implemented in two ways:

- A list of the conjuncts in the AND.
- An explicit AND operation.

These alternative ways of writing AND are illustrated by the following two equivalent rules:

```
(defrule rule1  "stock"
 (risk_tolerance high)
 (discretionary_income TRUE)
 =>
 (assert (investment stocks))
 (printout t "We recommend stocks." crlf)
)
(defrule rule1  "stock"
 (and (risk_tolerance high) (discretionary_income TRUE))
 =>
 (assert (investment stocks))
 (printout t "We recommend stocks." crlf)
```

```
)
```
**implementation of the OR operation**: The OR operation can be implemented by an explicit OR operation, i.e.,
```
(defrule rule3c  "high risk tolerance"
 (or  (now_own_stocks TRUE )(lottery_tickets TRUE ))
 =>
 (assert(risk_tolerance high))
)
```
Equivalently, one can write a separate rule for each disjunct in the OR:
```
(defrule rule3c1  "high risk tolerance 1"
 (now_own_stocks TRUE )
 =>
 (assert(risk_tolerance high))
)
(defrule rule3c2  "high risk tolerance 2"
 (lottery_tickets TRUE )
 =>
 (assert(risk_tolerance high))
)
```
Here is an actual CLIPS implementation. This implementation is a fairly straightforward translation of the generic KB1. More sophisticated implementations of KB1 would structure the knowledge base so that when sufficient information for a conclusion had occurred, the user would be spared extra questions.
```
;
; KB1 in CLIPS,  a demo rule based system
;
;
; Note: In the following knowledge base,
; we will use certain user interface functions
; which can be defined in CLIPS:
;
; yes-or-no-p asks a yes-no question
; ask-parm asks a fill-in question
; ask-parm asks a menu question
;
; To run this CLIPS knowledge base, you need these functions
; which are not shown here.
;
;
; INVESTMENT TYPE SUBSYSTEM
;
;    Rule 1: If "Risk tolerance" = high
;      AND "Discretionary income exists" = yes
;         then investment = stocks.
;
(defrule rule1  "stock"
 (risk_tolerance high)
 (discretionary_income TRUE)
```

```
 =>
 (assert (investment stocks))
 (printout t "We recommend stocks." crlf)
)
;
;   Rule 2: If "Risk tolerance" = low
;     OR "Discretionary income exists" = no
;        then investment = savings account.
;
(defrule rule2a  "savings account 1"
 (risk_tolerance low)
 =>
 (assert (investment "savings account"))
 (printout t "We recommend a savings account." crlf) )

(defrule rule2b  "savings account 2"
 (discretionary_income FALSE)
 =>
 (assert (investment "savings account"))
 (printout t "We recommend a savings account." crlf) )
;
; DISCRETIONARY INCOME SUBSYSTEM
;
;   Rule 5: If
;        ( Savings > $3000)
;        AND ("Do you own a boat" = yes
;          OR "Do you own a luxury car" = yes)
;        then "Discretionary income exists" = yes.
;
; First we will gather the information
;
(defrule rule5a  "boat"
 (not (has_boat ? ))
 =>
 (bind ?boat ( yes-or-no-p "Do you own a boat? " ))
 (assert (has_boat ?boat  ))
)
;
(defrule rule5b  "luxury car"
 (not (has_lux_car ? ))
 =>
 (bind ?lc ( yes-or-no-p "Do you own a luxury car? " ))
 (assert (has_lux_car ?lc  ))
)
;
(defrule rule5c  "savings balance"
 (not (savings_balance ? ))
 =>
 (bind ?sb ( ask-parm "What is your savings balance? " ))
 (assert (savings_balance ?sb))
)
;
; Now we will use the information determining discretionary income
```

```
;
(defrule rule5d  "has discretionary income"
 (savings_balance ?sb)
 (test( > ?sb 3000))
 (or (has_lux_car TRUE  ) (has_boat TRUE))
 =>
 (assert (discretionary_income TRUE))
)
;
;    Rule 6: If Savings <= $3000
;              OR
;              ("Do you own a boat" = no
;                AND "Do you own a luxury car" = no)
;              then "Discretionary income exists" = no.
;
(defrule rule6  "has no discretionary income"
 (savings_balance ?sb)
 (test( <= ?sb 3000))
 (and (has_lux_car FALSE  ) (has_boat FALSE))
 =>
 (assert (discretionary_income FALSE))
)
;
; RISK TOLERANCE SUBSYSTEM
;
;    Rule 3: If "Do you buy lottery tickets" = yes
;      OR "Do you currently own stocks" = yes
;         then "Risk tolerance" = high.
;
(defrule rule3a  "lottery tickets"
 (not (lottery_tickets ? ))
 =>
 (bind ?Lt ( yes-or-no-p "Do you purchase lottery tickets ? "))
 (assert (lottery_tickets ?Lt ))
)
;
(defrule rule3b  "currently own stocks"
 (not (now_own_stocks ? ))
 =>
 (bind ?s ( yes-or-no-p "Do you currently own stocks ? "))
 (assert (now_own_stocks ?s ))
)
;
(defrule rule3c  "high risk tolerance"
 (or  (now_own_stocks TRUE )(lottery_tickets TRUE ))
 =>
 (assert(risk_tolerance high))
)
;
;    Rule 4: If "Do you buy lottery tickets" = no
;      AND "Do you currently own stocks" = no
;         then "Risk tolerance" = low.
;
```

```
(defrule rule4  "low risk tolerance"
 (and (now_own_stocks FALSE)(lottery_tickets FALSE))
 =>
 (assert(risk_tolerance low ))
)
```

Figure 3.3.1:  KB1 Implementation

# 4. The Basic Proof Method

This chapter provides an overview of the basic method for formal proofs:

- Partition larger systems into small systems.
- Prove correctness on small systems by non-recursive means.
- Prove that the correctness of all these subsystems implies the correctness of the entire system.

## Introduction

An expert system is correct when it is complete, consistent, and satisfies the requirements that express expert knowledge about how the system should behave.

For real-world knowledge bases containing hundreds of rules, however, these aspects of correctness are hard to establish. There may be millions of distinct computational paths through an expert system, and each must be dealt with through testing or formal proof to establish correctness.

To reduce the size of the tests and proofs, one useful approach for some knowledge bases is to *partition* them into two or more interrelated knowledge bases. In this way the VV&E problem can be minimized.

## Overview of Proofs Using Partitions

The basic method of proving each of these aspects of correctness is basically the same. If the system is small, a technique designed for proving correctness of small systems should be used. If the system is large, a technique for partitioning the expert system must be applied and the required conditions for applying the partition to the system as a whole should be proven. In addition the correctness of any subsystem required by the partition must be ensured. Once this has been accomplished this basic proof method should be applied recursively to the subexpert systems.

To carry out a partitioning of an expert system, one generally requires expert knowledge to define the top level problem-solving strategy of the expert system. In Chapter 7, "Knowledge Modeling", a number of knowledge representations are outlined that may be useful in formalizing the top level structure of the knowledge base. Through knowledge acquisition with one or more expert, the top level structure of the knowledge base should be represented in a knowledge model. The correctness of this knowledge model should be validated with other experts or with standard reference materials in the target domain (the section in Chapter 9, on Validating the Semantic Consistency of Underlying Knowledge Items, addresses the problem of validating expert knowledge). When the formalization of the top level knowledge base has been so validated, the fact that the knowledge base has the validated structure can, from the standpoint of a formal proof, be assumed.

Once the top level structure of the knowledge base has been validated, to show the correctness of the expert system, the following criteria must be accomplished:

- Show that the knowledge base and inference engine implement the top level structure.
- Prove any required relationships among sub-expert systems or parts of the top level knowledge representation.
- Prove any required properties of the sub-knowledge bases.

Chapter 7, "Knowledge Modeling", discusses what exactly must be proved for various knowledge models and for various aspects of the correctness problem.

## *A Simple Example*

To illustrate the basic proof method, Knowledge Base 1 will be proved correct in Figure 5.1. Although this knowledge base is small enough to verify by inspection, the proof will be carried out in detail to illustrate the proof method.

**Knowledge Base 1**

Rule 1: If "Risk tolerance" = high
AND "Discretionary income exists" = yes
then investment = stocks.

Rule 2: If "Risk tolerance" = low
OR "Discretionary income exists" = no
then investment = "bank account".

Rule 3: If "Do you buy lottery tickets" = yes
OR "Do you currently own stocks" = yes
then "Risk tolerance" = high.

Rule 4: If "Do you buy lottery tickets" = no
AND "Do you currently own stocks" = no
then "Risk tolerance" = low.

Rule 5: If "Do you own a boat" = yes
OR "Do you own a luxury car" = yes
then "Discretionary income exists" = yes.

Rule 6: If "Do you own a boat" = no
AND "Do you own a luxury car" = no
then "Discretionary income exists" = no.

Figure 4.1: Knowledge Base 1

### *Step 1 -- Determine Knowledge Base Structure*

To prove the correctness of Knowledge Base 1 (KB1), the expert knowledge can determine that the system represents a 2-step process:

1. Find the values of some important intermediate variables, such as risk tolerance and discretionary income.

2. Use these values to assign a type of investment.

KB1 was built using this knowledge; therefore, it can be partitioned into the following pieces:

- A subsystem to find risk tolerance (part of Step 1).
- A subsystem to find discretionary income (part of Step 1).
- A subsystem to find type of investment given this information (part of Step 2).

To prove the correctness of a multi-step system, it must be proved that Step 1 satisfies the following criteria:

- For each set of inputs, all the outputs required by Step 2 are always produced by Step 1.
- For each set of inputs, all the outputs of Step 1 are single-valued.
- The correct outputs of Step 1 are assigned to each possible set of inputs.

It must also be proved for Step 2 that:

- For each set of inputs and computed Step 1 outputs, Step 2 produces some output.
- For each set of inputs and Step 1 outputs, all the outputs of Step 2 are single-valued.
- The correct outputs of Step 2 are assigned to each possible set of inputs and computed Step 2 outputs.

### *Step 2 -- Find Knowledge Base Partitions*

To find each of the three subsystems of KB1, an iterative procedure can be followed:

1. Start with the variables that are goals for the subsystem, e.g., risk tolerance for the risk tolerance subsystem.

2. Include all the rules that set subsystem variables in their conclusions. For the risk tolerance subsystem, Rules 3 and 4 are included.

3. Include all variables that appeared in rules already in the subsystem and are not goals of another subsystem.

4. For the risk tolerance subsystem, include "Do you buy lottery tickets" and "Do you currently own stocks".

5. Quit if all rules setting subsystem variables are in the subsystem, or else go to Step 2. For the risk tolerance subsystem, there are no more rules to be added.

Figure 4.2 below shows the partitioning of KB1 using this method.

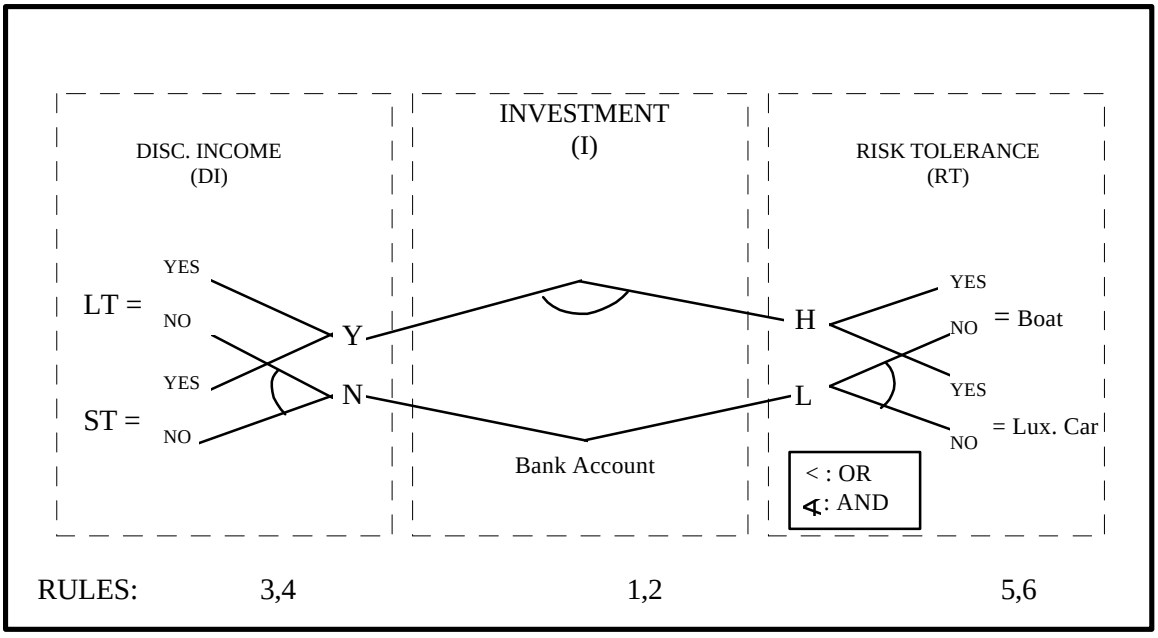



Figure 4.2: An Example of Knowledge Base Partitioning

## *Step 3 -- Completeness of Expert Systems*

**Completeness Step 1 -- Completeness of Subsystems**

The first step in proving the completeness of the entire expert system is to prove the completeness of each subsystem. To this end it must be shown that for all possible inputs there is an output, i.e., the goal variables of the subsystem are set. This can be done by showing that the OR of the hypotheses of the rules that assign to a goal variable is true.

For example, the discretionary subsystem of KB1 will be shown to be complete. The discretionary subsystem consists of these rules:

Rule 5: If "Do you own a boat" = yes

OR "Do you own a luxury car" = yes

then "Discretionary income exists" = yes.

Rule 6: If "Do you own a boat" = no

AND "Do you own a luxury car" = no

then "Discretionary income exists" = no.

Step 3.1: The first step is to form the OR of the possible outputs of the system:

"Discretionary income exists" = yes (4.1)

OR "Discretionary income exists" = no

(4.1) expresses the condition under which some conclusion is reached.

Step 3.2: For each output condition in (4.1), the user substitutes the OR of rule hypotheses for rules that imply that condition. For example, for

"Discretionary income exists" = yes (4.2)

the only rule inferring (4.2) is Rule 5; its hypothesis is:

"Do you own a boat" = yes (4.3)

OR "Do you own a luxury car" = yes

Since this is the only rule concluding (4.2), (4.3) is the OR of rule hypotheses implying (4.2).

Making the substitution of (4.3) for (4.2) in (4.1), and a similar substitution for:

"Discretionary income exists" = no (4.4)

the result is:

("Do you own a boat" = yes (4.5)

OR "Do you own a luxury car" = yes)

OR

("Do you own a boat" = no

AND "Do you own a luxury car" = no)

Step 3.3: Continue substitutions of the OR of rule hypotheses for inferred propositions (4.5) until the user obtains an expression where only input variables appear. In fact, (4.5) already contains only input variables, and no further substitutions are needed.

Step 3.4: Apply Boolean algebra to simplify the expression from Step 3; the goal is to show that the Step 3 expression always has the truth value TRUE.

Letting:

A = "Do you own a boat" = yes

B = "Do you own a luxury car" = yes

(4.5) can be rewritten as:

(A or B) or (Not A and Not B) (4.6)

Simplifying this gives:

(A or B) or (Not A and Not B)

= (A or B or Not A) and (A or B or Not B)

= true and true

= true

This means that the OR of conditions that imply *some* conclusion is true.

**Completeness Step 2 -- Completeness of the Entire System**

The results of subsystem completeness are used to establish the completeness of the entire system. The basic argument is to use results on subsystems to prove that successively larger subsystems are complete. At each stage of the proof there are some subsystems known to be complete; initially the subsystem that concludes overall goals of the expert system will be complete. At each stage of the proof, a subsystem that concludes some of the input variables of the currently-proved-complete subsystem is added to the currently complete subsystem. After a number of steps equal to the number of subsystems, the entire system can be shown to be complete.

When a complete subsystem that sets input variables of the currently complete subsystem is added to the latter, the augmented subsystem is complete. Any input to the augmented subsystem can be divided into a set V1 of input variables for the unaugmented system and a set V2 for the newly added subsystem. Note that some variables may be in both of these sets. Since the newly added subset is complete, given V1, that subsystem produces output O1. However, O1 union V2 is an input for the unaugmented system, which, because of its completeness, produces an output showing that the augmented system is complete.

Since the number of subsystems is finite, the process of augmentation ceases after a finite number of steps. By mathematical induction, using a similar argument to that of the previous paragraph, it follows that the entire system is complete.

For KB1, this result can be applied, or alternatively make the following specific argument: Inputs to the system as a whole can be partitioned into inputs for the risk tolerance and the discretionary income subsystems. Each of these is complete, and so produces a risk tolerance and discretionary income respectively. These are inputs to the investment subsystem its only inputs. Since the investment

subsystem is complete it produces an investment. So an output for the entire system exists for each input, and the system as a whole is complete.

### *Step 4 -- Consistency of the entire system*

The first step in proving the consistency of the entire expert system is to prove the consistency of each subsystem. To do this, the user must show that for all possible inputs, the outputs are consistent, i.e., that the AND of the conclusions can be satisfied.

For example, if an expert system concludes "temperature > 0" and "temperature < 100", the AND of these conclusions can be satisfied. However, if the system concludes, "temperature < 0" and "temperature > 100", the AND of these two conclusions has to be false. It is clear that based on the input that produced these two conclusions, it is not possible for all of the system's conclusions to be true at the same time and thus the system producing these conclusions is inconsistent.

**Consistency Step 1 -- Find the Mutually Inconsistent Conclusions**

The first step in proving consistency is to identify those sets of mutually inconsistent conclusions for each of the subsystems identified in the "Find partitions" step above.

Some sets of conclusions are mathematically inconsistent. For example, if a system describes temperature, the set:

{"temperature < 0", "temperature > 100"}

is mathematically inconsistent.

However, other conclusion sets that are not mathematically inconsistent may be inconsistent based on domain expertise. For example, one investment advisor expert system could be designed to recommend several types of investments to each investor (probably not a bad idea). For such a system, "investment = stocks" AND "investment = bank account" are not inconsistent; stocks and bank accounts are just two of the investments recommended for some investor. However, if the system were designed to recommend only one investment per investor, "investment = stocks" AND "investment = bank account" would be interpreted as a contradiction, and the system recommending this would be inconsistent.

Because some sets of conclusions are inconsistent because of domain expertise, finding all sets of inconsistent conclusions generally requires expert knowledge.

Note that if there are no mutually inconsistent conclusions in the expert system as a whole, then consistency is true by default, and no further consistency proof is necessary.

**Consistency Step 2 -- Prove Consistency of Subsystems**

If there are inconsistent conclusions in the knowledge base as a whole, then the next step in proving consistency is to prove the subsystems consistent. This can be done by showing that no set of inputs to a subsystem can result in any of the sets of inconsistent conclusions. For each set of inconsistent

conclusions, the user can construct, as detailed below, a Boolean expression B that represents all the conditions under which that set of inconsistent conclusions would be proved by the subsystem. If that Boolean expression can be shown to be FALSE, there are no such conditions.

Now the construction of the Boolean expression B to be proved false will be described. Let

S = {C1, ..., Cn}

be a set of potentially inconsistent conclusions for one of the subsystems.

B will be constructed by a backward chaining process, starting with

B0 = C1 AND ... AND Cn

Let Ci be one of the Cs. For all rules that conclude Ci, construct the OR of these rules initial conditions. Then substitute the resulting expression into B0.

Continue these substitutions until an expression results that has only the inputs to the expert subsystem. For each atomic Boolean expression A that is the conclusion of a rule in the subsystem, substitute the OR of the rule if parts of rules that conclude A. After at most a finite number of such substitutions, the user obtains an expression that states when all the C's would be true in terms of the input variables of the subsystem.

For the risk subsystem, the only inconsistent set of rule conclusions is:

S = { "Risk tolerance" = high and "Risk tolerance" = low }

The only initial conditions for "Risk tolerance" = high is from Rule 3:

"Do you buy lottery tickets" = yes

OR "Do you currently own stocks" = yes

and the only initial conditions for "Risk tolerance" = low is from Rule 4:

"Do you buy lottery tickets" = no

AND "Do you currently own stocks" = no

Let:

A0 = ("Do you buy lottery tickets" = yes)

A1 = ("Do you currently own stocks" = yes).

This means:

not A0 = ("Do you buy lottery tickets" = no)

not A1 = ("Do you currently own stocks" = no).

Using this notation:

B0 = (A0 OR A1) AND (NOT A0 AND NOT A1)

For this small subsystem, B0 is actually expressed in terms of inputs to the subsystem (i.e., B0 is actually B).

Distributing the top level AND over the OR,

B0 = (A0 AND (NOT A0 AND NOT A1))

OR (A1 AND (NOT A0 AND NOT A1))

The first subexpression is FALSE because it contains A0 AND NOT A0. Likewise, the second is FALSE because it contains A1 AND NOT A1. Therefore, B0 is FALSE because it is the OR of only FALSE expressions.

**Consistency Step 3 -- Consistency of the Entire System**

The results of subsystem consistency are used to establish the consistency of the entire system. The basic argument is to use results on subsystems to prove that successively larger subsystems are consistent. At each stage of the proof, there are some subsystem known to be consistent; initially, this is the subsystem that concludes goals of the expert system as a whole. At each stage of the proof, a subsystem that concludes some of the input variables of the currently-proved-consistent subsystem is added to the currently consistent subsystem. After a number of steps equal to the number of subsystems, the entire system can be shown to be consistent.

When a consistent subsystem that sets input variables of the currently consistent subsystem is added to the currently consistent subsystem, the augmented subsystem is consistent. Any input to the augmented subsystem can be divided into a set V1 of input variables for the unaugmented system and a set V2 for the newly added subsystem. Note that some variables may be in both of these sets. Since the newly added subset is consistent, given V1, that subsystem produces and output O1. However, O1 union V2 is an input for the unaugmented system producing output due to its consistency. This shows that the augmented system is consistent.

Since the number of subsystems is finite the process of augmentation ceases after a finite number of steps. By mathematical induction, using the above mentioned argument, it follows that the entire system is consistent.

For KB1, one can apply the result, or alternatively make the following specific argument: Inputs to the system as a whole can be partitioned into inputs for the risk tolerance and the discretionary income subsystems. Each of these is consistent, and so produces a consistent set of risk tolerance and discretionary incomes, respectively. These are inputs to the investment subsystem, and are that system's only inputs. Since the investment subsystem is consistent, it produces a consistent investment. Thus an output for the entire system exists for each input, and the system as a whole is consistent.

The other subsystems of KB1 can be proved consistent in the same way.

### *Step 5 -- Specification Satisfaction*

In order to prove that KB1 satisfies its specifications, the user must actually know what its specifications are. This is a special case of the general truth that in order to verify and validate, the user must know what a system is supposed to do. Specifications should be defined in the planning stage of an expert system project.

To illustrate the proof of specifications it will be assumed that KB1 is supposed to satisfy:

> A financial advisor should only recommend investments that an investor can afford.

As with many other aspects of verification and validation, expert knowledge must be brought to bear on the proof process. For KB1, an expert might say that anyone can afford a savings account. Therefore, the user only has to look at the conditions under which stocks are recommended. However, that same expert would probably say that just having discretionary income does not mean that the user can afford stocks; that judgment should be made on more than one variable. Therefore, it would be reasonable to conclude that KB1 does not satisfy the above specification.

However, if the expert does agree that the expert system observes all necessary inputs, one must use inputs to the expert system to express a specification. For KB1, this means that the specification is reexpressed as:

> KB1 recommends stocks only when there is discretionary income.

The user can prove this for the investment subsystem by assuming:

> NOT discretionary income

and proving:

> NOT stocks

The only rule that concludes stocks has "discretionary income" = yes in an AND in its "if" part. Therefore, the investment system satisfies the specification.

To prove the entire system satisfies the specifications, the user must look at the conditions under which "discretionary income" = yes is concluded from inputs for the system as a whole. A financial expert would surely say that owning a luxury car or boat does not mean that discretionary income actually exists and the system as a whole fails the specification, an expected outcome of a small example system tackling a complex subject.

# 5. Finding Partitions Without Expert Knowledge

This chapter presents techniques for partitioning large expert systems when expert knowledge is unavailable.

## Introduction

Generally, it is best to partition a knowledge base using expert knowledge, resulting in a knowledge base that reflects the expert's conception of the knowledge domain. This, in turn, facilitates communication with the expert, and later maintenance of the knowledge base. Chapter 7, "Knowledge Modeling", presents techniques for partitioning using expert knowledge.

Sometimes, however, it is not possible to obtain expert insight into a knowledge base. In this case functions and incidence matrices can be extracted from the knowledge base, and the information contained therein used to partition the knowledge base.

## Functions

### *Expert Systems are Mathematical Functions*

Expert systems are, among other things, complicated functions in the mathematical sense of function. [By definition, a function is a set F of ordered pairs, such that if (a,b) and (c,d) are in F, and a = c, then b = d.] Less formally, a function is a single-valued mapping from an input space (called the domain) to an output space (called the range); i.e., there is only one value of the function for each point in the input space. For example, KB1 is a function that for each set of user data (i.e., amount of savings, personal property, etc.) assigns a type of investment.

The input variables to an expert system viewed as a function are the variables that are not computed inside the expert system, but are asked the of user or looked up in a data base. Variables that are inferred by rules or computed by functions in the knowledge base are not input variables. In KB1, for example, purchase of lottery tickets and ownership of boats and luxury cars are input variables, while risk tolerance and discretionary income are not. Tolerance and discretionary income, however, are inputs to the investment subsystem of KB1.

Propositions that are possible conclusions of the expert system are Boolean output variables of the expert system. Numerical or enumerated variables that are considered outputs of the expert system are also output variables. When viewed as a function the value of an expert system is a vector of these individual output variables.

## *Partitioning Functions into Compositions of Simpler Functions*

Functions can be written as compositions of simpler functions. For expert systems, two of the important relations that build more complex functions from simpler ones are Cartesian product and function composition.

**Cartesian Product**

Suppose that an expert system made two different kinds of recommendations, e.g., a traffic management system that both set the timing of lights and controlled access to exit ramps. This expert system could be considered as a function E that computed light timing and on ramp access from certain inputs, e.g.:

E(inputs) = ( timings, access).

E could be split into two expert systems that computed these results separately:

E = ( timings(inputs), access(inputs)) (5.1).

While some of the inputs and intermediate conclusions might appear in both subsystems, (5.1) decomposes E into two subsystems using the Cartesian product operation. The Cartesian product operation in this case takes the two separate conclusions, timings(inputs) and access(inputs) and builds the conclusions of E:

( timings(inputs), access(inputs))

by putting the separate conclusions of the subsystems together in a fixed, predetermined order.

More generally, if:

(y1,...,ym ) = f(w1,...,wk),

(z1...,zq ) = g(x1,...,xn,),

then:

(y1,...,ym, z1...,zq ) = f(w1,...,wk) X g(x1,...,xn,),

where X is the Cartesian product operator.

Applied to expert systems, this result means that if there is an expert system where input Ws are used to compute the conclusion Ys, and the Xs are used to compute the Zs, the system can be partitioned into subsystems:

(y1,...,ym ) = f(w1,...,wk),

(z1...,zq ) = g(x1,...,xn,),

and the results concatenated together.

**Function Composition**

Function composition uses the results of an earlier function A as the inputs to a later function B to compute a single overall function C. This overall function is the result of :

1. Starting with the inputs to A.

2. Applying the function A to these inputs.

3. Applying B to the results of Step 2.

4. Using the results of step 3 as the value of C.

In the Pavement Maintenance Expert System (PAMEX), for example, various data items are used to compute the "Pavement Serviceability Index" (PSI) and other measures of pavement life. The PSI and other similar parameters are then fed into a follow-up set of rules that choose appropriate maintenance procedures. PAMEX can be considered as a composition of the subsystem that computes indices with the subsystem that uses these to compute appropriate maintenance procedures.

In mathematical notation, suppose the output of an expert system depends on a set of variables, y1,...ym, i.e.:

E = f(y1,...,ym)

In addition, suppose each of the y's is a function of some other variables, i.e.,:

yi = gi(x1,...,xmi)

Then E = f( g1(x11,...,x1m),

g2(x21,...,x2m),

....

gn(xm1,...,xnm))

i.e., the expert system E is the result of applying the function f to the result of applying Gs to the input variables.

Note that which variables are functions of which others are properties of the expert system. This means that a function implemented by an expert system can not be arbitrarily rewritten as the composition of simpler functions. Instead, the choice of simpler functions is motivated by:

- Which variables are functions of which other ones in the expert system knowledge base.
- Which rewriting of the function computed by an expert system as the composition of functions reduces the size of the VV&E problem.

For KB1, investment is a composition of an investment function with risk tolerance and discretionary income functions:

- investment( risk_tolerance( "lottery tickets", "stock ownership"),
- discretionary_income( "boat", "luxury car" ) ).

## Dependency Relations

To find the functions embedded in a knowledge base, it is helpful to compute the dependency relation among variables.

### *Immediate Dependency Relation*

The first step is to compute the immediate dependency relation. If X1 and X2 are variables in the knowledge base, X2 is immediately dependent on X1, if and only if, the following are true:

- X1 appears in an expression that computes X2.
- X1 appears in the if part of a rule that sets or concludes X2.
- X1 is an input to a function that computes X2.

The table below shows the immediate dependency relation for Knowledge Base 1. A1 appears in cell (I,Jj), if and only if, variable J is immediately dependent on variable I.

The immediate dependency relation for Knowledge Base 1 is shown in table 5.1.

Table 5.1: Immediate Dependency Relation for KB1

| immediate dependency | LC | B | S | LT | DI | RT | INV |
|---|---|---|---|---|---|---|---|
| luxury car (LC) | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| boat (B) | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| stocks (S) | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| lottery tickets (LT) | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| discretionary income (DI) | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| risk tolerance (RT) | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| investment (INV) | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

The immediate dependency relation shows which variables influence the value of other variables through one level of computation (one rule inference or function computation) in the expert system.

## *Computing the Immediate Dependency Matrix*

The immediate dependency matrix can be computed by syntactic inspection of the source code (including both rules and procedures) of the expert system knowledge base. Although the underlying computation is basically the same, the computation can be described either as a database or as a sparse matrix computation.

## *Database Description of Immediate Dependency Computation*

The immediate dependency matrix can be constructed directly as follows. In this construction, the matrix is represented by a relation with 2 colums:

- Column 1: A variable that affects another variable.
- Column 2: A variable that is affected by another variable.

Each row in the table represents a pair of variables such that the first affects the second directly in some rule or function.

Start with an empty database.

For each rule or function in the knowledge base, find all pairs (x,y) such that x is an input and y an output of the rule or function. Put each such pair in the database.

It is also possible to construct the data base as the composition of two simpler tables:

An input table:

- Column 1: An input variable.
- Column 2: A rule or function in the knowledge base.

A row (x,f) appears in this table when a variable x is an input to a rule or function f.

An output table:

- Column 1: An output variable.
- Column 2: A rule or function in the knowledge base.

A row (x,f) appears in this table when a variable x is an output to a rule or function f.

Now by applying the following join operation to the tables, build atable where:

- Column 1 is an input variable.

- Column 2 is an output variable.

There is a row for each variable pair (x,y) such that for some f, (x,f) is in table 1 and (y,f) in table 2.

## *Sparse Matrix Description of Immediate Dependency Computation*

The relation between input and output variables describes a aparse matrix representing the immediate dependency relation. The rows and columns are indexed by variables. A 1 appears for the matrix position described by each row in the table constructed in the preceeding section, and a 0 appears for all other matrix positions. By the definition of the immediate dependency relation, this sparse matrix represents that relation.

The join-based computation described above can be written using sparse matrices as follow:

1. Construct input and output matrices:

   The input matrix is based on table 1. The rows are indexed by variables and the columns by functions and rules. A 1 appears when a variable is an input to a rule or function. Zeros fill the other matrix positions.

   The Output matrix is based on table 2, but is the transpose of the matrix that directly represents table 2. The rows are indexed by functions and rules. The columns are indexed by variables. A 1 appears when a variable is an output of a rule or function. Zeros fill the other matrix positions.

2. Compute the product of the input matrix by the output matrix.

3. Booleanize the product matrix, i.e. replace all non-zero entries by 1s.

This product matix has a 1 at position (x,y) whenever the product has a non-zero, i.e. when there is a rule or function f where x is an input to f and f has y as an output.

## *An Example*

**Dependency Relations of Rules on Variables in Knowledge Base 1**

In KB1, all atomic formulas set by the knowledge base are of the form:

VARIABLE = VALUE

When this is the case, the immediate dependency of variables and rules is sufficient to obtain the dependency among variables. Table 5.2 shows how variables influence rules.

Table 5.2: How Variables Influence Rules

| | R1 | R2 | R3 | R4 | R5 | R6 |
|---|---|---|---|---|---|---|
| LC | | | | | 1 | 1 |
| B | | | | | 1 | 1 |
| S | | | 1 | 1 | | |
| LT | | | 1 | 1 | | |
| DI | 1 | 1 | | | | |
| RT | 1 | 1 | | | | |
| INV | | | | | | |

**Dependency Relations of Variables on Rules in Knowledge Base 1**

Table 5.3 shows how rules influence variables.

Table 5.3: How Rules Influence Variables

| | LC | B | S | LT | DI | RT | INV |
|---|---|---|---|---|---|---|---|
| R1 | | | | | | | 1 |
| R2 | | | | | | | 1 |
| R3 | | | | | | 1 | |
| R4 | | | | | | 1 | |
| R5 | | | | | 1 | | |
| R6 | | | | | 1 | | |

**Dependency Relations of Variables on Variables in Knowledge Base 1**

Multiplying A*B creates the matrix showing how each variable influences others. Positive numbers in cell (R,C) indicate that the variable in row R influences the variable in column C. Making this into a Boolean matrix yields the immediate dependency matrix for variables in KB1.

Table 5.4 shows the immediate dependency matrix for KB1.

Table 5.4: Immediate Dependency Matrix for KB1

| | LC | B | S | LT | DI | RT | INV |
|---|---|---|---|---|---|---|---|
| LC | 0 | 0 | 0 | 0 | 2 | 0 | 0 |
| B | 0 | 0 | 0 | 0 | 2 | 0 | 0 |
| S | 0 | 0 | 0 | 0 | 0 | 2 | 0 |
| LT | 0 | 0 | 0 | 0 | 0 | 2 | 0 |
| DI | 0 | 0 | 0 | 0 | 0 | 0 | 2 |
| RT | 0 | 0 | 0 | 0 | 0 | 0 | 2 |
| INV | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Using the extended immediate dependency relation R just defined, the user can compute a sub-knowledge-base that is sufficient to compute a set of variables. Let SO be a set of output variables for a function f, chosen as discussed in the previous section. Let RR be either one of the R *a n or the relation R *d. Then the sub-knowledge base that computes f is defined by:

x is in Sub_KB(f) iff x RR y for some y in SO.

## *Operations on Relations*

Using the immediate dependency relation, one may compute the influences of variables through any number of levels of inference or function computation and composition. This requires union and composition relations defined as follows:

**Relation:** A relation is, from a mathematical standpoint, a set of ordered pairs.

For example, the immediate dependency relation is shown as an ordered pair in figure 5.1:

{(LC,DI), (B,DI), (S,RT), (LT,RT), (DI,INV), (RT,INV)}

A pair (x,y) appears in the immediate dependency relation if and only if x influences the value of y.

Figure 5.1: Immediate Dependency Relation as Ordered Pairs

**Domain**: If R is a relation {x| for some y, xRy} is the *domain* of R. Some examples of domains are shown in figure 5.2.

Domain of the investment subsystem of KB1:

{ ("discretionary income" = yes, "risk tolerance" = high),

("discretionary income" = no, "risk tolerance" = high),

("discretionary income" = yes, "risk tolerance" = low ),

("discretionary income" = no , "risk tolerance" = low )}

Domain of the immediate dependency relation for KB1:

{luxury car, boat, stocks, lottery tickets, discretionary

tolerance, risk tolerance, investment}

Figure 5.2: Examples of Domains

**Range:** {y| for some x, xRy} is the range of r. For example, the range of the investment subsystem of KB1 is { stocks, savings account}; the range of the immediate dependency relation is {0, 1}.

**Composition:** If R1 and R2 are relations, the relation (R1 o R2) is defined as follows: x (R1 o R2) z if and only if there is a y such that x R1 y and y R2 z.

For example, the composition of the immediate dependency relation of KB1 with itself is:

{(LC,INV), (B,INV), (S,INV), (LT,INV)}.

For an immediate dependency relation R among the variables of an expert system, (x,z) is in RoR if and only if there is a y such that (x,y) and (y,z) are in R; i.e., there is a variable y such that x influences y and y influences z. In other words, RoR shows the variables that indirectly influence another variable acting through a single intermediate variable.

**Matrix representation:** When range(R1) = domain(R2)

the composition operation R1 o R2 can be computed by matrix multiplication. A relation R is represented by a matrix M = {m(i,j)} if and only if:

m(i,j) = 1 iff x R y where x is variable i and y is variable j

m(i,j) = 0 otherwise.

Table 6.1 shows the immediate dependency relation in matrix form.

If Mi represents Ri, B( M1 o M2) represents R1 o R2, where:

M1 o M2 represents matrix product of M1 and M2.

B(M) = {bm(i,j)} represents the Boolean operation on matrices, i.e.,

bm(i,j) = 1 iff m(i,j) != 0

bm(i,j) = 0 iff m(i,j) = 0.

**Theorem 5.1:** If R1 and R2 are immediate dependency matrices, B(M1 o M2) represents R1 o R2 when M1 represents R1 and M2 represents R2.

This theorem says that the representation of the indirect dependency relation with one intermediate variable can be computed by Booleanizing the matrix product of the immediate dependency matrix with itself.

**Proof:** Let M be the matrix that represents R1 o R2, based on a numbering of the relevant variables v1,...vn. The (i,j) entry of M is 1 if and only if vi influences vj. This means that there two sets of inputs where the vi's differ, and also where the results of applying (R1 o R2) to these inputs differ. On these two inputs, one of the inputs to R2 must vary on the two inputs; if no input to R2 varied, the output would also not vary on the two inputs.

Since at least one input variable to R2 varies when vi varies, let vk be such an input to R2. Since vk varies when vi varies, R1(i,k) = 1. Likewise, since vj varies when vk varies, R2(k,j) = 1. This means that:

the kth entry of row i = 1

the kth entry of column j = 1.

As a result, kth summand in the inner product:

(Row i of M1) * (column j of M2) (5.2)

is 1. Since all entries of M1 and M2 are non-negative, the Cartesian product (6.2) is non-zero. This means that (M1 o M2) has a non-zero (i,j) entry, so B( M1 o M2)(i,j) = 1. The result is that everywhere M is 1, B(M1 o M2) is also 1.

Now let (m,n) be a location in B( M1 o M2) which is 1. This will be true only if the (m,n) entry of M1 o M2 is non-zero. Since all entries of M1 and M2 are non-negative, (M1 o M2)(m,n) > 0. This entry of M1 o M2 is the inner product:

(row m of M1) * (column n of M2)

so the inner product is positive. This is possible only if there is a k so that the kth entry in each of these vectors is non-zero. This means that for some k, the kth entry of row m of M1 and the kth entry of column n of M2 are both 1, i.e.,:

M1(m,k)=1

M2(k,n)=1.

This means that vm influences vk and vk influences vn. Therefore, vm influences vn, showing that M, the representation of (R1 o R2), has a 1 wherever B( M1 o M2) has a 1.

Combined with the earlier result, it is evident that the two matrices M and B(M1 o M2) have the same set of 1's. Since both matrices have only 1 and 0 entries, the matrices are equal.

For example, in KB 1, B influences DI, as indicated by the 1 in the (B,DI) entry of the immediate dependency relation of KB1. In table 6.1, this appears in the (2,5) location. Likewise, DI influences INV, and the (5,7) entry of the table is 1, meaning that multiplying the table by itself, when the inner product of row 2 by column 7 is computed, the 1's in position 5 cause the inner product to be non-zero. This represents the fact that variable 2 (B) influences INV, variable 7, through the intermediary of variable 5, VI.

Table 5.5 shows the matrix product of the immediate dependency relation by itself. In this case, it is also the Boolean composition operation.

Table 5.5: Matrix Product of the Dependency Relation by Itself

| immediate dependency | LC | B | S | LT | DI | RT | INV |
|---|---|---|---|---|---|---|---|
| luxury car (LC) | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| boat (B) | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| stocks (S) | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| lottery tickets (LT) | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| discretionary income (DI) | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| risk tolerance (RT) | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| investment (INV) | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

**Power:** If R is a relation,

R**1 = R

R**(n+1) = R o (R**n).

The power relation finds those variables which influence a variable through a chain of intermediate variables of some particular length. For R**n the chain of intermediate variables is of length n-1.

If M represents R and M**n is the product of n Ms, then B(M**n) represents R**n.

The previous table shows R**2 when R is the immediate dependency relation. Higher powers of the immediate dependency relation are empty (all zeros in the matrix representation).

**Theorem 5.2:** M**n represents the indirect influence of variables with n-1 intermediate variables.

**Proof:** Theorem 5.2 follows from Theorem 5.1 by mathematical induction.

**Union:** If R1 and R2 are relations with the same domain and range, the relation (R1 U R2) is the relation such that x (R1 U R2) y iff x R1 y or x R2 y.

The union and composition operations are used to build relations about dependency through multiple levels of inference. For example, if x D2 y, if and only if x influences y, directly or through an intermediate variable, D2 = D U D o D, where D is the intermediate dependency relation and o is the composition operation.

**Theorem 5.3:** If Mi represents Ri, B(M1+M2) represents R1 U R2.

**Proof:** B(M1+M2)(i,j) = 1 iff M1(i,j) or M2(i,j). Iff x is the ith variable and y is the jth variable, M1(i,j) or M2(i,j) iff x R1 y or x R2 y, i.e.

x (R1 U R2) y.

Figure 5.2 represents:

R U (R**2)

where R is the immediate dependency relation of KB1.

**Accumulation:** The accumulation operator R *a n is defined as follows:

R *a 1 = R

R *a (n+1) = (R *a n) U (R ** (n+1))

The accumulation R *a n of a relation finds all the variables that influence a variable through a chain of n-1 or fewer intermediate variables.

**Theorem 5.4:** R *a n represents the dependency relation between n-1 or fewer intermediate variables. If M represents R, B(M *a n) represents R *a n.

**Proof:** This follows from Theorems 5.2 and 5.3.

**Dependency:** The relations { lim R *a n } form an increasing sequence of relations, i.e., if (x,y) is in *a n, (x,y) is in *a m for m >= n. Therefore, the limit of this sequence as n --> infinity exists, and is equal to the union of the R *a n for all n. This limit will be called R*d.

Define the dependency relation D(R) as follows: x D(R) y iff the variable x influences the variable y. It is only possible for x to influence y if there is some (possibly empty) chain of intermediate, e.g., x, z1, ..., zn, y such that each variable influences its successor, i.e., each successive pair of variables is in the relation R. However, then x R**(n+1) y, so x (R *a n) y,

so x R*d y, and D(R) <= R*d.

However, if x R*d y, for some n, (x,y) R*a m for m > n (by definition of limit). Pick an m0 > n. Then x R*a m0 y, so for some m1 <= m0,

x R**(m1) y. Then there is a chain of m1+1 intermediate variables, z1,...zm1+1 such that x,z1,...,zm1+1,y is a sequence in which successive variables are in R, and R*d < D(R).

Combining this with the previous result proves theorem 5.5.

**Theorem 5.5:** The limit R*d of the accumulation relations represents the dependency relation D(R).

Since both the sequences {B(M*n)} and {R*n} are monotone increasing and have only a finite number of possible values, each of these sequences is eventually constant. That constant is the limit of the sequence. Pick an n0 great enough so that each sequence has reached its limit. By Theorem 5.4, B( M *n0 ) represents R *n0 where M represents R. Since equal matricies represent equal relations, the limits can be substituted in this "represents" relation, proving

**Theorem 5.6:** The matrix lim(n->infinity)(B(M *a n)) represents D.

The dependency relation represents the relation that is true for all variables that influence a given variable, and false otherwise. Figure 5.2 is the accumulation of the immediate dependency relation of KB1. An entry in the table is 1 iff the variable on the right is dependent on a variable on the left.

To compute the dependency relation from the immediate dependency relation:

- Compute in sequence each R *a n.
- When the R *a n no longer change, the current R *a n is the dependency relation R*d.

Table 5.6. shows the dependency relation of the immediate dependency relation of Knowledge Base 1.

Table 5.6: Immediate Dependency Relation of KB1

| | LC | B | S | LT | DI | RT | INV |
|---|---|---|---|---|---|---|---|
| luxury car (LC) | 0 | 0 | 0 | 0 | 1 | 0 | 1 |
| boat (B) | 0 | 0 | 0 | 0 | 1 | 0 | 1 |
| stocks (S) | 0 | 0 | 0 | 0 | 0 | 1 | 1 |
| lottery tickets (LT) | 0 | 0 | 0 | 0 | 0 | 1 | 1 |
| discretionary income (DI) | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| risk tolerance (RT) | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| investment (INV) | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

## Finding Functions in a Knowledge Base

To carry out a partition of a knowledge base based on function composition, it is necessary to find functions embedded in the knowledge base. In particular, the goal is to find subsets SI and SO of the knowledge base variables such that the:

- Values of SO are a function of the inputs in SI.
- Variables in SI are used at most infrequently outside this function.

### *Choosing the Output and Input Variables of a Function*

Each column vector in the dependency relation matrix shows which variables influence each other. For example, the first 4 columns of the dependency matrix for KB 1 are all 0s, because these are input variables and are not influenced by any other variables in the KB. Discretionary income (DI) has 1's for the two variables that influence it, namely the boat and luxury car. Investment has nearly all 1's, because all variables except itself influence its value.

To find the set of variables whose Cartesian product will be the output of a function in the KB, cluster via high correlation the column vectors in the table. The clusters should be performed in such a way that all members of a cluster are highly correlated with each other, indicating that all the variables computed by a function use about the same set of input variables.

The variable clusters of the dependency relation of the immediate dependency relation of Knowledge Base 1 are:

{luxury car, boat}

{stocks, lottery tickets}

{discretionary income, risk tolerance}

{investment}

Once a set of output variables has been chosen, the set of input variables for the function consists of the union of all variables for each member of the output variable set. Table 5.7 shows variable clusters of the dependency relation of KB1.

Table 5.7: Variable Clusters of the Dependency Relation of KB1

| VARIABLE CLUSTER | INPUT VARIABLES |
|---|---|
| {LC, B, S, LT} | none |
| {DI} | {LC,B} |
| {RT} | {LT,S} |
| {INV} | {DI,RT} |

### *Finding the Knowledge Base that Computes a Function*

In the previous section, the input and output variables were computed for a set of functions that partition the knowledge base. Table 5.4 illustrates this partitioning for knowledge base 1.

Given the input and output variables for a function, the subset of rules and functions in the knowledge base used to compute that function can be found as follows. Note that the input and output matrices from which the immediate dependency relation is computed are used in this computation. Refer to **Computing the Immediate Dependency Relation** for details about computing these matrices.

1. Start with the output variables of the function. Set the current unprocessed output variables to the set of output variables. Start with an empty set of rules and KB functions in the KB subset implementing the function; call the set of implementing and rules IMP.

2. For each current unprocessed output variable y, and each function or rule f which has y as an output, add f to IMP. Remove y from the set of unprocessed output variables.

3. For each f added to IMP, examine all x such that x is an input to f. If x is not an input to the function for which a KB is being computed, add x to the set of unprocessed output variables.

4. Continue this process until the set of unprocessed output variables is empty.

## Hoffman Regions

For logical completeness and consistency of an expert system, an important concept is the Hoffman regions (suggested by Roger Hoffman of FHWA). If V1...Vn are the variables of a knowledge base, with domains D1...Dn respectively, a Hoffman region is a maximal subset of the input space, the Cartesian product D1x...xDn, on which each atomic formula in the knowledge base has a single truth value. For any knowledge base, there is a unique set of Hoffman regions that cover and partition the input space.

A run of an expert system is completely determined by the values of the atomic formulas that appear in the KB rules. Provided that the expert system does not use external numerical software, there is no

need to run two different test cases that evaluate the same on all the atomic formulas. If two different test cases evaluate some atomic formula differently, however, the firing of some rule, and hence the results of the expert system, may differ between the two test cases. Therefore, the set of test cases that must be tested are in 1-to-1 correspondence with the regions where all the atomic formulas have the same value. These regions where the atomic formulas are the same are called Hoffman regions.

Each point in input space determines truth values for each of the atomic formulas in the knowledge base. A relation H(P1,P2) can be defined on input point spaces as follows: H(P1,P2) is true if and only if P1 and P2 determine the same set of atomic formula truth values for all atomic formulas in the KB. H so defined is an equivalence relation, and partitions the input space into mutually disjointed regions that cover the input space.

It is generally not possible to find simple, exact descriptions for all the Hoffman regions when a knowledge base contains atomic formulas that contain several variables, e.g., exp(X)<Y^3. It is possible, however, to find an approximate set of Hoffman regions of descriptions such that:

- Every Hoffman region is in the approximate set of Hoffman regions.
- A member of the approximate set of Hoffman regions is either a Hoffman region, or is the empty set, i.e. is an empty region of input space.

The set of possible Hoffman descriptions D can be computed as follows:

- For atomic formulas containing two or more variables, the Hoffman regions of these atomic formulas are TRUE and FALSE.
- Sort all the atomic formulas containing only one variable into subsets, putting all the formulas containing the same variable together.
- Normalize formulas containing relation operators so that the variable appears on the left.
- Lexically sort the formulas for each variable as follows:
  - The major sort is by the right side of the formula.
  - The minor sort is by relational operator, where the relation operators in ascending order are: <, <=, =, >=, >.
- Create a set of intervals for each numerical variable that:
  - Cover the real line, or at least the possible domain of the variable.
  - For all points in any interval, the truth values of the atomic predicates (of that single variable) are the same.
  - The intervals are maximal, given the truth value constraint.
- For each string variable, let the Hoffman regions be the list of values that appear in the KB.
- Let the Hoffman regions of the KB as a whole be the Cartesian product of the Hoffman regions for the individual variables.

Note that in KB's with atomic formulas with more than one variable, the use of TRUE or FALSE as the Hoffman regions is a compromise to avoid having to decide exactly when combinations of these formulas are true. This means that some Hoffman regions may be unsatisfiable. Therefore, if

exhaustive testing shows an inconsistency in some Hoffman region which is partly defined by atomic formulas of more than one variable, there are two possibilities:

- The Hoffman region is unsatisfiable, so the expert system is OK.
- The Hoffman region is satisfiable, and the expert system has an inconsistency.

If a Hoffman region is found where the expert system is inconsistent, it should be determined whether the Hoffman region is satisfiable. Table 5.8 illustrate this concept.

Table 5.8: Hoffman Regions for KB1

| | | | |
|---|---|---|---|
| LC=yes<br>B=yes<br>LT=yes<br>S=yes | LC=yes<br>B=yes<br>LT=yes<br>S=no | LC=yes<br>B=yes<br>LT=no<br>S=yes | LC=yes<br>B=yes<br>LT=no<br>S=no |
| LC=no<br>B=yes<br>LT=yes<br>S=yes | LC=no<br>B=yes<br>LT=yes<br>S=no | LC=no<br>B=yes<br>LT=no<br>S=yes | LC=no<br>B=yes<br>LT=no<br>S=no |
| LC=yes<br>B=no<br>LT=yes<br>S=yes | LC=yes<br>B=no<br>LT=yes<br>S=no | LC=yes<br>B=no<br>LT=no<br>S=yes | LC=yes<br>B=no<br>LT=no<br>S=no |
| LC=no<br>B=no<br>LT=yes<br>S=yes | LC=no<br>B=no<br>LT=yes<br>S=no | LC=no<br>B=no<br>LT=no<br>S=yes | LC=no<br>B=no<br>LT=no<br>S=no |

## *When is a Partitioning Advantageous*

Let CH(KB0) be the cardinality of the Hoffman region set of knowledge base KB0. The worst case in proving a result on a knowledge base KB with sub-KB KB1 is, using the result of the previous section, CH(KB1) + CH(~KB1). If this number is significantly smaller than CH(KB), the partitioning pays off in reducing the size of a VV&E problem.

**Hoffman Regions of Partitioned KB1**

The KB can be split into the following pieces:

- **Final conclusion KB:** This contains rules 1 and 2, and determines the type of investment.
- **Risk tolerance KB:** This contains rules 3 and 4, and determines the comfort level of the client regarding risk.
- **Discretionary income KB:** This contains rules 5 and 6, and determines whether the client has discretionary income.

Each of these KB's has two input variables each with two values, or four Hoffman regions. Therefore the total number of Hoffman regions after partitioning is twelve, a 25 percent reduction. A greater reduction is found in many larger knowledge bases.

# 6. Knowledge Modeling

This chapter presents some *knowledge models* that can be used to partition knowledge bases using expert knowledge. The chapter includes:

- Definition of knowledge models.
- Using knowledge models for VV&E.
- Using knowledge models in the expert system lifecycles.
- Some example knowledge models.
- Proof techniques for specific knowledge models.
- Specific knowledge models

Appendix A presents some mathematical results used in the chapter about partitioning using the clear box methodology.

## Introduction

Knowledge models are high level templates for expert knowledge. Examples of knowledge models are decision trees, flowcharts and state diagrams. By organizing the knowledge, a knowledge model helps with VV&E by suggesting strategies for proofs and partitions; in addition, some knowledge models have mathematical properties that help establish completeness, consistency or specification satisfaction.

More particularly:

- The knowledge model highlights the main points of a knowledge base, often obscured in the knowledge base.
- A knowledge model partitions a large KB into smaller, easier to verify, pieces.
- There are mathematical properties of the knowledge model that help establish the correctness of a knowledge base.

### *An Example of a Knowledge Model*

PAMEX (Pavement Maintenance Expert System) is an expert system for pavement maintenance management [Aougab et. al., 1988]. A top level model of PAMEX consists of a partition of the problem space on the following three variables:

- Level of information about the pavement; the 3 values are extensive, some and little or none.
- Range of pavement serviceability index (PSI); the 3 values are above 2.8, between 2.8 and 2.0, and below 2.0.
- The level of treatment desired; the 3 values are long-range, mid-term and short-term.

For each of the twenty seven regions formed by the Cartesian product of the three regions on each variable, there is a small expert system that handles problems in that region. These small expert systems use the same pavement variables, i.e., PSI and other more specific pavement measurements. In this case, the model is a decision tree, discussed and illustrated in the next section.

**Using Knowledge Models in VV&E**

The steps in using a knowledge model in VV&E are:

- Collect the knowledge model from:
  - The domain expert(s) working on the project.
  - Standards documents in the domain.
  - Notes from knowledge acquisition at the time an existing system was built.
- Validate the knowledge; see Chapter 9 on knowledge validation for details. This step is to ensure that the knowledge going into the expert system represents correct expert knowledge.
- Prove the expert system using the knowledge model is complete, consistent and satisfies its specifications; this chapter, as well as chapters on partitioning and small systems, provides information on how to develop these proofs.

# Decision Trees

## *Introduction*

A decision tree is a set of decisions that partitions the input space into a set of disjoint regions that cover the entire input space. In a decision tree system, a sequence of decisions based on user input and other data are used to classify the input problem before going on to the rest of problem solution.

The top of the decision tree corresponds to the start of the decision process. At each interior node of a decision tree, the problem is supposed to be assigned to one and only one of the subnodes. The solution of the detailed problems is often handled by specialized expert systems tailored to the specialized situations found by the decision tree.

## *Definition*

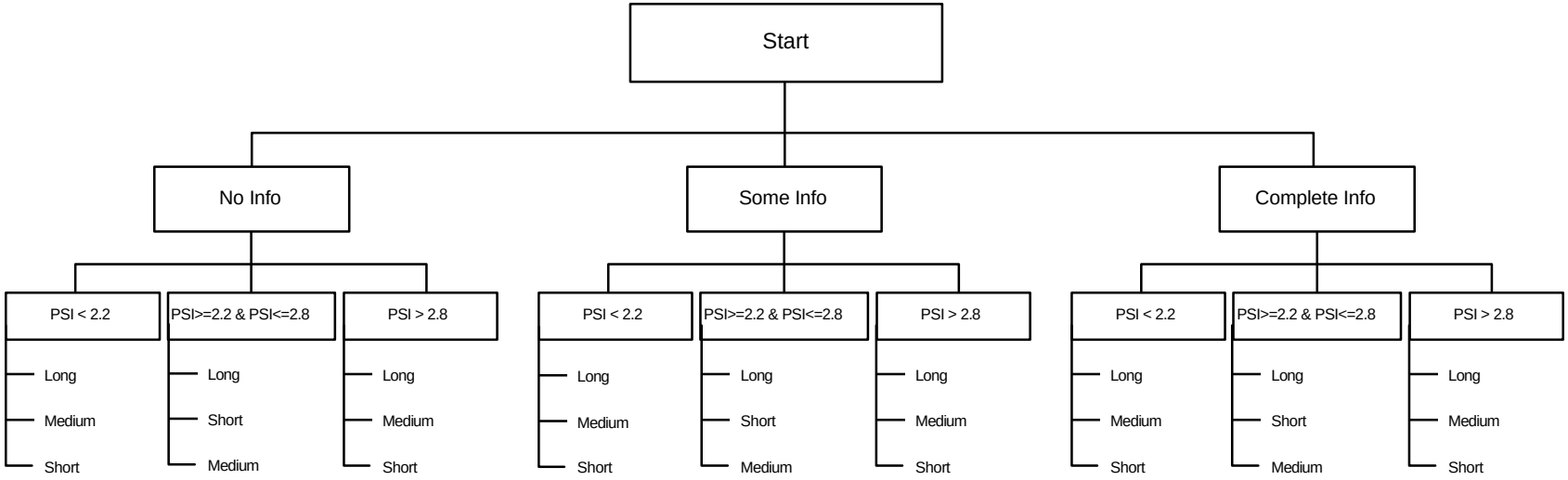
A decision tree expert system has a structure that is described by a tree. A decision tree system has the following properties:

- Each interior node of the tree has a variable or expression assigned to it.
- Each edge to a subtree is labeled with a set of values for that variable or expression on the parent node.
- All possible values of a variable are on some edge.
- No variable value is on two different sibling edges.
- Associated with each leaf node is a subsystem or output(s). A subsystem at a tip node N of a decision tree is called to solve the problems for which variables appearing in the tree have values associated with the path that leads to N.

### *Example*

A decision tree for PAMEX is illustrated in figure 7.1 of the following page.

# PAMEX Decision Tree




**LEGEND**

PSI: Pavement Serviceability Index

Info: the amount of information available about the pavement

short, medium, long term: the time period for which the fix is made, subject to budget constraints

Pavement Maintenance Expert System

Figure 6.1: Pamex DT

### *Use During Development*

Decision trees are a useful way to organize expert knowledge. Their use is indicated when the expert can describe in what order information is obtained and used to partially determine a solution. Drawing a decision tree from information the expert(s) have provided is a good way to present the knowledge engineer's conception of the information back to the domain expert for validation.

### *Use During VV&E*

To model an expert system as a decision tree for the purpose of showing correctness, the following conditions should be satisfied:

- Each possible set of inputs should be in one and only one of the partitions generated by the decision tree.
- For each partition, there is an expert system (a subsystem of the entire system) that correctly solves problems in that partition.
- Experts validate the decision tree.
- The expert system assigns each input to the correct partition as the result of a finite computation.

To prove *completeness* of an expert system modeled by a decision tree, prove the following:

- Each possible problem in the input space is assigned to some partition of the decision tree.
- Each expert system assigned to one of the partitions computes a solution for each problem assigned to it.

To prove *consistency* of an expert system modeled by a decision tree, prove the following:

- Each possible problem in the input space is assigned to at most one partition of the decision tree.
- Each expert system assigned to one of the partitions computes at most one solution for each problem assigned to it.
- Each computed solution is internally consistent.

To prove *satisfaction of a requirement* of an expert system modeled by a decision tree, it needs to be shown that the requirement is satisfied for the expert system associated with each tip of the decision tree.

## Ripple Down Rules

### *Introduction*

Ripple down rules (RDR's) [Kang, et al, 1994.] are a special case of decision trees for reasoning with defaults. RDR's are guaranteed to be complete and consistent.

## *Definition*

With ripple down rules, the knowledge base is organized as lists of rules. If the conditions ("if" part) of a rule are satisfied, then the expert system moves to the part of the knowledge base attached to this rule. In some cases, this is another list of rules. If so, the expert system tests the rules in the sublist. If there is no sublist of rules, or if none of the sublist rules are satisfied, then the conclusions "then part" of the rule is used. Figure 6.2 demonstrate an example of a small expert system for vehicles classification.

## *Example*

As an example, a small expert system for vehicles classification is presented.

The main list is:

L1.1: If NOA (Number -of Axles) is 2
Try List 1-1; Default = Car.

L1.2: If NOA is 3,
Try List 1-2; Default = 3 Axle-single unit Truck.

L1.3: If NOA is 4,
Try List 1-3; Default = 4 Axle-single unit Truck.

L1.4: If NOA is 5,
Try List 1-4; Default = 5 Axle-single unit Truck.

etc.

Here are the lists that fill out the next level of the knowledge base; note that this is not an exhaustive knowledge base.

L1-1.1: If S1 <= 12, it is a Car-Van-Pick up.

L1-1.2: If S1 <= 20, it is a 2 Axle-single unit Truck.

L1-1.3: If S1 > 20, it is a 2 Axle Bus.

L1-2.1: If S1 <= 12 & 8 < S2 <= 18, it is a Light Vehicle w/ Single Axle Trailer.

L1-2.2: If 7 < S1 <= 20 & S2 <= 8, it is a 3 Axle-singular unit Truck.

L1-2.3: If S1 > 20 & S2 <= 8, it is 3 Axle Bus.

L1-2.4: If Else, it is a 2 Axle Tractor w/ Singular Axle Trailer.

L1-3.1: If S1 > 7 & S2 + S3 <= 12, it is a 4 Axle-singular unit Truck.

L1-3.2: If S1 > 7 & S2 <= 8 & S3 > 6, it is a 3 AxleTractor w/ Singular Axle Trailer.

L1-3.3: If Else, it is a 2 Axle Tractor w/ Tandem Axle Trailer.

L1-4.1: If S2+S3+S4 < 16, it is a 5 Axle-singular unit Truck.

L1-4.2: If S2 <= 8 & S4 <= 10.5, it is a 3 Axle Tractor w/ Tandem Axle Trailer.

L1-4.3: If S2 > 8 & S3 + S4 <= 12, it is a 2 Axle Tractor w/ Tridem Axle Trailer.

L1-4.4: If S2 > 8 & 12 < S3 + S4 <= 16, it is a 2 Axle Tractor w/ Tridem Axle Trailer Split.

etc.

Figure 6.2: Example ES (continued)

Similar rule lists could expand lists 1-3 and 1-4.

The expert system starts the example and the system moves to list 1-2 (likewise for the other L1 rules). If none of the entry conditions to the rules in list L1 is satisfied, the default of L1, car, is the KB conclusion.

Under the condition that NOA is 3, the system moves to list 1-2 and if none of the entry conditions to those rules is satisfied, the default of L2, axle-single unit truck, is the KB conclusion.

## *Use During Development*

Kang et al., 1994 point out that it is possible to add correction rules to a running ripple down rules expert system. Whenever an error occurs, that error gets added to the last list of rules which the system tried before choosing an erroneous default.

Ripple down rule systems are ideally suited to problems where knowledge has the following structure:

- Early decisions made on a problem narrow the range of possible solutions, while later decisions pick particular solutions from a selected class.
- There is a default solution at each stage of the solution process.

**Changing a Ripple Down Rule System**

Ripple down rules are a special type of decision tree. For a knowledge base that consists of a series of more detailed decisions, but where the bases of the more detailed decisions vary for different points of the decision tree, the ripple down rules model is appropriate.

Given: an RDR, and a rule (if C then A) which the algorithm should execute, the algorithm *change* modifies the KB to make (if C then A) part of the system:

case 1: Top level list of RDR is empty.

If default(RDR) = A, do nothing,

else insert (if C then A) as a 1-element list of RDR.

case 2: The conditions on the first rule in the top level list of RDR = C.

Attach to the first rule the RDR with default = A and empty rule list.

case 3: The conditions on the first rule subsume C.

Replace the RDR attached to the first rule, denoted by R2, with *change*(R2).

case 4: C subsumes the conditions on the first rule.

Replace the first rule with (if C then A).

case 5: C and the conditions of the first rule can be simultaneously satisfied.

Insert (if C then A) before the first rule.

otherwise: Let RDR = H++T, where H is the first rule in the top level list, and T is the rest of the rules. Insert (if C then A) in T.

### *Use During VV&E*

*Completeness* of a RDR system follows from the following theorem:

**A Ripple-Down-Rule System is Complete.**

Proof: Note that part of an RDR system attached to a top level rule is itself an RDR system.

Define the level of an RDR system as follows: If the system has only 1 rule list, it is of level 1. If the system has N+1 rule lists, then it has level 1+Max(level of RDR subsystems of the top level rule list).

Let R be an RDR system of level N+1. Assume all RDR systems of level N are complete. For any input, either some top level condition is satisfied or not. In the latter case, the system concludes the default. In the former case, the system finds the conclusion computed by RDR rules from the first satisfied top level rule. If there is a rule list associated with that condition, the conclusion is from an RDR system of level at most N, and so exists. If there is no rule list, the conclusion is from the condition itself. Therefore an RDR system produces a conclusion in all cases.

In a similar way, it can be proven that all RDR systems are *consistent.* Consistency, however, requires an additional check: that the conclusions associated with each path through the ripple down rule tree are consistent.

*Satisfaction Of Specification*: To verify that an RDR satisfies a proposition P:

1. Verify or modify the default of the top level rule set.
2. Verify or modify the first rule, if any in the top level list to satisfy P.
3. Verify or modify the RDR system attached to the first rule, if any.
4. Let RDR = H++T, where H is the first rule in the top level list, and T is the rest of the rules. Verify or modify T to satisfy P.

*Generalizations Of RDR*: A generalization of RDR systems occurs when the conditions in RDR rules are replaced with specialized expert systems whose purpose is to make the decision specified in the if

part of the RDR rule. When, in an ordinary RDR system an RDR rule if part is evaluated, a generalized RDR system may call an expert subsystem. This is a backward chaining process although RDR systems are more structured than general backward chaining systems.

The same algorithms for VV&E on RDR systems also work for generalized systems, provided that the expert subsystems carry out the tests provided in the rule condition that the subsystem replaces.

## State Diagrams

### *Introduction*

A state diagram is a useful formal representation for the top level of process control expert systems.

### *Definition*

A state diagram system is one where there is a unique *state* at every step of a solution, and at each state, there is a function that determines the next state.

### *Example*

A state diagram can be used to model driver behavior on a road segment. A set of *states* indicates the situation and/or goal of the driver. For example, some possible states are:

- Distance ahead too small.
- Clear road ahead.
- Approaching desired exit.

A driver model based on these states is shown below. The case statement branches on the value of the variable *state*.

```
state = start_loop;
while ( state is not equal to exit )
case (state)
[
 case start_loop:
        if (distance ahead is too small)
                state = distance ahead too small;
        else (approaching desired exit)
                state = exit;
        else (clear road ahead)
                state = clear road ahead;
        else delay a small time increment;
 case clear road ahead:
        if (current speed < desired speed)
                increment speed;
        delay a small time increment;
```

```
        state = start_loop;
 case distance ahead too small:
        if (current speed < desired speed)
                { if (passing possible)
                        pass;
                  else decrease speed; }
        delay a small time increment;
        state = start_loop;
 case exit;
        return any current useful information to calling program
}
```

In this example, the decision to pass may be made by another expert system. In addition, fuzzy logic is often used to assign a membership grade representing how much the current situation belongs to each of the possible states. In this case, the expert system chooses a state with the highest membership grade and executes the code associated with that state.

State Diagram Systems Represented as Rules: Systems based on state diagrams may be encoded into expert system rules. The following include two of the rules that would implement the above example in rule form:

```
if state = start_loop
        and distance ahead is too small
        then state = distance ahead too small.
if state = start_loop
        and approaching desired exit
        then exit and return information to calling program
if state = start_loop
        and clear road ahead
        then state = clear road ahead
if state = start_loop
        and not ( distance ahead is too small
                or approaching desired exit
                or clear road ahead)
        then delay a small time increment
if state = clear road ahead
        and current speed < desired speed
        then increment speed
                and delay a small time increment
                and state = start_loop;
if state = clear road ahead
        and current speed >= desired speed
```

```
        then and state = start_loop;
if state = distance ahead too small
        and current speed < desired speed
        and passing possible
        then pass
                and delay a small time increment
                and state = start_loop
if state = distance ahead too small
        and current speed < desired speed
        and not passing possible
        then decrease speed
                and delay a small time increment
                and state = start_loop
if state = distance ahead too small
        and current speed >= desired speed
        then decrease speed
                and delay a small time increment
                and state = start_loop
```

### *Use During Development*

State diagram models are useful during development when expert knowledge has the following characteristics:

- The problem solution consists of a series of distinct steps.
- Which step to choose is a complex, but knowledge-based decision.
- The possible paths through the steps may contain loops.

To run such a rule-based system based on state diagrams generally requires an inference engine that can do both forward and backward chaining with the same knowledge base in a strategy called forward chaining with local backward chaining. In this strategy applied to the knowledge base forward chaining keeps applying rules until a rule containing the command to exit the knowledge base fires. Backward chaining is used to establish the conditions within the rules, e.g., passing possible in the above example.

### *Use During VV&E*

*Completeness* of a state diagram system can be established by showing that for any inputs the system eventually reaches a *final* state where it returns information and exits to the calling environment. In a complex system in which the predicates that control transitions between states are themselves expert systems, the proof of completeness is hierarchical:

1. Assume that the expert subsystems satisfy their specifications. Using this information, prove that the system reaches a final state.

2. Prove that the expert subsystems satisfy their specifications, and also that they terminate for any possible inputs.

Since a table of one value for each of a set of variables is consistent, state diagram systems that return a set of variable values when they reach a final state are *logically consistent.* The set of variable values may be unsatisfiable, however, given the specifications for the expert system and expert knowledge about the domain.

To show that the output of a state diagram system satisfies a specification for the expert system demonstrate that:

- For each state, if the specifications are satisfied on entering the state, they are also satisfied when leaving the state.
- The specifications are satisfied at the start state. Often the specifications are trivially satisfied at the start state, because the values of output variables are unknown.
- The system always reaches a final state.

*Satisfaction Of Specifications*: To prove that a specification for a state diagrams is satisfied, one should prove that for any input in the input set of the specification, the state diagram eventually reaches a final state in which the requirements of the specification are satisfied.

# Flowcharts

## *Introduction*

Flowcharts are another method for recording expert knowledge and can serve as a model for the knowledge in an expert system.

## *Use During Development*

Flowcharts can be implemented best by using a procedural programming language, i.e., a language that permits:

- Blocks, i.e., sequences of statements used as a single statement.
- Branching statements, e.g., if-then-else or switch statements.
- Loops, e.g., while, do and for loops.
- Function calls, permitting a procedure to call other procedures or itself.

If, however, some procedural knowledge is included in a largely non-procedural knowledge base and the available implementation shell does not permit procedural programming, it may be more convenient to encode the procedural knowledge in rules.

In this case, a flowchart can be represented in rule form by associating a state with each box in the flowchart and by writing rules that describe the transitions between boxes represented by the lines in the flowchart.

### *Use During VV&E*

Completeness, consistency, and satisfaction of specifications for flowcharts are similar to the problems for state diagrams.

If the effect of the flowchart is to set variable values, slot values on objects, or build other data structures, the logical statements represented by these structures can usually be satisfied. The result of the flowchart is logically consistent but not necessarily consistent with the specifications for the expert system or other expert knowledge about the application domain.

*Consistency:* Flowcharts need not produce consistent output even when:

- The flowchart always reaches an exit box.
- All of the variables that are outputs of the system have a unique value.

If, however, all possible tuples (ordered list of variables) of output variable values are consistent i.e., for any assignment of values to output variables, it is logically possible, and consistent with domain expertise, for the variables to have those variables simultaneously. Then, if for all inputs, the flowchart defines unique values for all the output variables, the flowchart is consistent.

*Completeness*: A flowchart is logically complete, if no matter what the inputs, the flowchart always reaches an exit box. For this to be true, the one must prove that:

- The computation eventually exits from any loop entered within the flowchart.
- All functions called within the flowchart satisfy their specifications for all inputs and perform their computation in a finite time.

*Satisfaction Of Specifications:* To show that a flowchart system satisfies its specifications, the basic strategy is to show that if the specifications are satisfied on entry to each box in the flowchart, they are satisfied on exit from the box. Specifications are generally satisfied before the initial box because variables are not yet set to values, but indicating that specifications are satisfied at the start is a necessary part of the proof of specifications.

If box A of the flowchart has just one exit line L going to box B, then A, B, and L represent a sequence of separate computations. To show that this part of the flowchart satisfies the specifications, one should demonstrate that:

- The computations in A and B can always be carried out in only finite time.
- If the specifications are satisfied on entry to A and B, they are satisfied on exit. In proving the specifications for B the user can assume the results of the computations in A, in addition to the specifications that were assumed on entry to A.

If box A of the flowchart performs a test to decide a proposition P, and if A has exits to box B if P is true and box C if P is false, then the user must demonstrate that:

- The specifications are true at the exit box(es) when starting at B with the assumption of the specifications plus P, and that the computation always reaches an exit box in a finite computation.
- The specifications are true at the exit box(es) when starting at C with the assumption of the specifications plus not P, and that the computation always reaches an exit box in a finite computation.

If a flowchart contains a loop, one must demonstrate that, for all inputs satisfying the specifications, the following criteria are met:

- The specs are true on exit from the loop.
- Given the following assumptions at the loop exit:
    - The specifications.
    - The results of computations in the loop.
    - The conditions for exit from the loop.

    The flowchart computation reaches an exit box in finite time and the specifications are true when reaching the exit box.

## Functionally Modeled Expert Systems

### *Introduction*

As discussed in the chapter on partitioning without expert knowledge (see Chapter 5), an expert system can be thought of as a *function*. A function maps sets of inputs (information the expert system receives from the user or other external sources) into a set of outputs reflecting actions taken and conclusions inferred by the expert system. Ideally, the function that an expert system represents is that which maps each set of problem inputs into the set of actions and inferences that an expert would make given those inputs. The expert system will be said to *implement* this function, and the function will be said to *model* the expert system with the understanding that an expert system only approximates the behavior of an expert.

Some functions are built from simpler functions with operations such as (function) composition or Cartesian product (operations discussed in more detail below). Sometimes, because of domain knowledge, the expert system should represent a function that is constructed from simpler functions. If

that is the case, the structure of the function provides the knowledge engineer with tools for structuring and partitioning an expert system.

More particularly, the operations of Cartesian product and function composition in the category of functions are of particular importance in modeling expert systems. Let E be an expert system such that the output of E involves setting variables O1,...,On such that the values of the O's are independent of each other. Then E implements the Cartesian product of functions fi such that Oi = fi(Ii), where Ii is a subset of the inputs of the entire expert system found by computing the dependency relation (see Chapter 6 on partitioning without expert knowledge) starting with Oi.

If one of the fi is a composition of functions, e.g.

$$fi = h( g1(Ii), ..., gm(Ii))$$

then using the same techniques of Chapter 6, one can find subsystems of the original expert system that implement the g's and h can be found.

As discussed in more detail below, if the expert subsystems are complete, consistent and satisfy specifications, *and* if there is consistency and specification satisfaction among independently chosen possible values of Cartesian component subsystems, the entire expert system is complete, consistent and satisfies specifications.

Note that this does *not* mean that completeness, consistency and specifications satisfaction of arbitrary subsets of an expert system imply corresponding results about systems as a whole. The subsets *must* be those that implement functions used to construct the function that models the expert system, *and* certain additional requirements among the outputs of component systems must be met.

Expert knowledge is generally of great benefit in identifying:

- Independent outputs that can be used to decompose an expert system into a product of expert systems.
- Intermediate hypotheses that are functions of the problem inputs but are themselves inputs to a later function that produces some or all of the outputs of the system as a whole.

Following are some examples of composite functions which provide opportunities for structuring and partitioning expert systems.

## *Use During Development*

These strategies often simplify development by replacing a single development task with two or more, which is less than the original task. ***During VV&E***, these strategies likewise replace a single VV&E task with two or more development tasks where the total size is less than the original task.

In each of these cases, the key to whether the partitioning makes these problems smaller is found by counting Hoffman regions. If E is partitioned into E1,...,En, then if:

$$(H(E1)+...+H(En)) / H(E)$$

is significantly less than 1, partitioning E into the Ei decreases the size of the development or VV&E problem. Note that usually, *some* rules and variables may be contained in more than one of the Ei.

Cartesian Product Systems: Sometimes an expert system E is required to make more than one decision, e.g., to find values for two different (sets of) variables. In this case, the user can represent the expert system function e of input I as:

e(I) = (e1(I), ..., en(I)).

Using the techniques of chapter 7, the user can find subsystems Ei which implement ei respectively. If H(X) is the number of Hoffman regions in expert system X, then if

(H(E1)+...+H(En)) / H(E)

is significantly less than 1, partitioning E into the Ei decreases the size of the VV&E problem. [Note that *some* rules and variables generally appear in more than one Ei.]

*consistency*: If each of the Ei is consistent, *and* if the union of consistent sets of output from each of the Ei is consistent, the entire expert system is consistent.

*completeness*: If each of the Ei is complete, the entire expert system is complete.

*specification satisfaction*: Generally, proving that specifications are satisfied will involve consideration of the interaction of the outputs of the Ei. However, if a specification is of the form

If C1 and C2 ... and Cn then S (6.1)

then (6.1) is equivalent to the set of specifications

If (AND Ei satisfies Ci) then S.

Final Layer Partitioning: In final layer partitioning, the expert system is partitioned into:

- *The final layer expert system* that consists of all rules and functions that have as their direct outputs conclusions of the knowledge base.
- *Information gathering expert subsystems* that conclude the inputs to the final layer system.

The final layer system contains all rules and functions that produce one or more of the conclusions of the entire expert system. The inputs of the final layer expert system are the inputs to these rules and functions. In KB1, the investment subsystem is the final layer expert system.

For each of the input variables to the final layer expert system, there is an expert system that determines that input to the final level; that expert system can be found using the methods in the chapter on partitioning without expert knowledge. In particular, if the final level input variables are v1,...,vn, let E1,...,En be the expert systems that set these variables.

Those Ei and Ej which overlap greatly, so that:

(H(Ei) + H(Ej)) / H( Ei union Ej) >= 1

should be combined into a single expert system that produces both vi and vj. If, on the other hand,

(H(Ei) + H(Ej)) / H( Ei union Ej)

is significantly less than 1, Ei and Ej should be kept separate. Note that as described in the chapter on partitioning without expert knowledge, clustering of vectors from incidence matrices can be used to determine which of the information gathering subsystems to combine.

Partitioning into a final layer subsystem and information gathering subsystems is particularly useful when there are many rules which compute outputs from the information gathered from the subsystems. PAMEX is an example of such an expert system. In this case, incompleteness or inconsistency in the final layer expert system causes the same error in the entire expert system; furthermore, if there are many rules in the final layer subsystem, such errors are easy to make.

*Consistency*: The entire expert system is consistent when:

- The final layer expert system is consistent whenever it gets consistent inputs.
- Each of the information gathering subsystems is consistent.
- All unions of consistent output from each of the information gathering subsystems are consistent.

*Completeness*: If each of the information gathering subsystems is complete and the final layer expert system is complete, then the entire expert system is complete.

*Satisfaction Of Specifications*: Generally, proving that specifications are satisfied will involve consideration of the interaction of the outputs of the information gathering subsystems.

However, if a specification is of the form:

If C1 and C2 ... and Cn and Cf then S (6.2)

where Ci is a condition on subsystem Ei and Cf is a condition on the final layer,

then (6.2) is equivalent to the set of specifications:

If (AND Ei satisfies Ci)

and the final layer satisfies Cf,

then S is satisfied.

Intermediate Variables: Intermediate variables are variables that are computed or inferred from input variables, and are used to infer or compute conclusions.

Many expert systems can be decomposed into two sequential steps (an expert can often tell the user about such a decomposition):

1. Determine the value of some intermediate variables.

2. Draw conclusions from these intermediate variables.

In addition, an intermediate variable is useful for partitioning only if some of the input variables of the system as a whole are used for computing the intermediate variable.

In function notation, an expert system with an intermediate variable is of the form:

e(x1,...,xn) = e(x1,...,xk, y), where y = g(xk+1,...,,Xn).

Results about completeness, consistency, and specification satisfaction are entirely analogous to those for final level partitioning. However, the role of the final level expert system is that expert system which implements the function:

e(x1,...,xk, y)

with inputs x1,...,xk and y. This expert system can be found by the method in chapter 5. The single information gathering subsystem is:

g(xk+1,...,Xn).

Partitioning Of The Function Domain: Let E be an expert system which implements the function e(I), where I is a vector of inputs. Let the domain of I be some domain D, such that D is partitioned into mutually exclusive subsets {Di}, i.e.,

Union{Di} = D

Di intersection Dj = NULL for i != j

Let Ei be the expert system that implements the function:

e restricted to Di

Then the following results relate correctness of E to the correctness of the Ei.

*Consistency*: If each of the Ei is consistent, so is E.

*Completeness*: If each of the Ei is complete, so is E.

*Satisfaction Of Specification*: If a specification is satisfied by each Ei, it is satisfied by E.

Examples of domain partitioning occur in decision tree systems. The effect of the decision tree is to partition the entire domain of the expert system into subsets, each of which satisfies the conditions along the path from some leaf node of the decision tree to the root of that tree.

# Verifying Knowledge Model Implementations

## *Overview*

Knowledge models are useful for proofs because knowledge models may have certain established properties, such as consistency and completeness, that automatically apply to any system that uses the knowledge model. This means that one can simplify the task of proving something about an expert system by showing that it uses a knowledge model.

However, nothing is free. If one uses a knowledge model to establish the properties of a system, one must show that the system actually uses, i.e. implements, the knowledge model. This requirement is explained below.

## *Implementation of a Knowledge Model*

An expert system implements a knowledge model if:

- The data required by the knowledge model can be identified in the expert system
- The data used in the knowledge model is interpreted by the expert system according to the rules required by the knowledge model.

For example, to show that a rule-based expert system implements a decision tree, it should be shown that:

1. The expert system has rules that fire for each branch of the decision tree
2. The expert system gathers the information needed to select a branch in the decision tree
3. After gathering that information, the expert system selects the branch, i.e. the expert systems the subsystem attached to the branch determined by the just-gathered information.

## *Proofs Using a Knowledge Model*

If a knowledge model is used to establish that an expert system has some property, there are two things that need to be done:

1. Show that for a knowledge base that fits the knowledge model, the desired property is true.
2. Show that the expert system implements the knowledge model. How to do so is the subject of this section.

## *EXAMPLE*

Verifying a System based on Decision Tables

The KB1 demonstration expert system is shown in Figure 4.1. The information in the knowledge base is shown in the following decision tables.

Example Decision Tables:

Investment Decision Table

| | | | | |
|---|---|---|---|---|
| Risk Tolerance | yes | yes | no | no |
| Discretionary Income | yes | no | yes | no |
| Investment | Stocks | Bank Account | Bank Account | Bank Account |

Discretionary Income Decision Table

| | | | | |
|---|---|---|---|---|
| Boat | yes | yes | no | no |
| Luxury Car | yes | no | yes | no |
| Discretionary Income | yes | yes | yes | no |

Risk Tolerance Decision Table

| | | | | |
|---|---|---|---|---|
| Lottery Tickets | yes | yes | no | no |
| Stocks | yes | no | yes | no |
| Risk Tolerance | yes | yes | yes | no |

### *Analyzing KB1 With These Decision Tables*

To illustrate verifying a knowledge base, the knowledge base expressed in the decision tables will be accepted as correct; our current goal is to see that the code implementing the knowledge base contains the information in the decision tables, and only that information.

The following tables shows which rules in Figure 4.1 implement which parts of the decision tables.

| **Rule** | **Table** | **Columns** |
|---|---|---|
| 1 | Investment | 1 |

| | | |
|---|---|---|
| 2 | Investment | 2 thru 4 |
| 3 | Risk Tolerance | 1 thru 3 |
| 4 | Risk Tolerance | 4 |
| 5 | Discretionary Income | 1 thru 3 |
| 6 | Discretionary Income | 4 |

To illustrate how this table is interpreted, Row 2 means that Rule 2 implements columns 2 through 4 in the Investment decision table above.

## *Building the Rule/Decision Table Relation*

The Rule/Decision-Table relation was created by inspection, but the information therein is actually the result of simple mathematical reasoning that need not be done in detail, but which must be doable. In particular, it must be shown that whenever the conditions of one of the decision tree columns associated with a rule are true, the rule produces the conclusions(s) of that column of the decision tree. For example, choosing column 2 in the Investment decision table means that:

Risk Tolerance = yes

Discretionary Income = No

This causes Rule 1 to fail and rule 2 to succeed, producing the results of column 2 of the decision table.

It must also be shown that the only way for any rule to fire is to satisfy some column listed for it in the Rule/Decision Table relation. For Rule 2 this follows from the definition of OR, which requires that one of its arguments is true.

The combination of the these two kinds of arguments show that the rules contain the same logical information as the decision tables. However, it is also necessary to show that the expert system actually uses these rules for each branch of the decision tree. This is because it is possible that the inference engine might never fire a rule that would succeed if it were fired. Therefore, to show that an expert system actually implements the decision tree, one must show that the inference engine gathers the necessary information, and fires the right rules.

## *Verifying and Implemented Expert System Code*

To illustrate this process, it will be shown that the implementation code in Clips, shown in Step 3.2 of the Handbook finish this example, implement decision trees similar to those shown above. [The Clips code uses an additional criterion of amount of savings for discretionary income, so the decision trees do not exactly apply to its knowledge base.]

First, we show that Clips gathers the information needed to run the decision tables. Rules a-5c and 3a-3b gather this information. These rules contain only conditions in their if parts that are satisfied when Clips starts, so these rules will be fired, since Clips is forward chaining.

Here we are using properties of Clips described in the User Reference, and are assuming that Clips meets its specifications. This means that we are proving that our knowledge base is correct, assuming Clips meets the specifications we use in the proof. This makes our knowledge base correct conditional on the correctness of Clips, but this is a reasonable compromise in practice. It is much more likely that something new will contain an error than a well-used program like Clips. However, it should be noted that errors have been found in much simpler library programs than Clips, and that in safety-critical systems, the assumptions made about Clips should be verified by testing.

Given that the rules 5a-5c and 3a-3b fire, information needed to put the current problem being run on Clips into some columns of the risk tolerance and discretionary income tables is gathered. This information creates conditions under which 5d and 6 can fire, according to conditions in the rule/decision table relation table. This determines the value of discretionary investment. Similarly, rules 3c and 4 have enough information to be fired by Clips' forward chaining inference engine. This provides the information needed to fire the rules in the investment subsystem (rules 1 and 2). As a result, in all situations, the relevant rules fire. The determination of which rules fire under various decision table conditions is determined by the rule/decision table relation constructed using rules 1, 2, 3c, 4, 5d, and 6. Comparison of these rules with the decision tables, as illustrated above, complete the proof that the Clips system implements the decision tables.

## *Verifying a System based on State Diagrams*

The State Diagram Relation

Following is a table that represents the example state diagram implemented in procedural pseudocode in the VVE Handbook:

| **State:** | **Actions** | **Condition** | **Actions** | **Next State** |
|---|---|---|---|---|
| start: | _ | | | |
| start | | distance ahead too small | _ | distance too small |
| start | | approaching desired exit | _ | approaching desired exit |
| start | | clear road ahead | _ | clear road |
| start | | default | small delay | start |
| clear road: | _ | current speed < | increment speed, | start |

| | | | |
|---|---|---|---|
| | desired speed | small delay | |
| clear road | default | small delay | start |
| distance too small | desired speed and passing possible | pass, small delay | start |
| distance too small | default | decrease speed, small delay | start |
| exit: return info to calling program | | | |

The state diagram table has the following meaning:

STATE : ACTIONS names the state and lists actions that are taken when the state is entered. For example, exit : return info to calling program means that on entry to the state exit, return info to calling program is executed. When there is no action to be executed, The dash (-) is used after the state name when there is no action to be executed.

CONDITIONS denotes the entry conditions for a path from the current state. For example, the entry condition for the 2nd. path from the start state is distance too small. Paths from a state are tried in the same order as listed in the table. default may be used for the last path from a state to indicate that that path is always taken if none of the earlier paths are.

ACTIONS denotes the actions taken once the conditions for a path have been satisfied. These are actions that are to be performed when transitioning between a particular pair of states. For example, the actions increment speed, small delay are performed when taking the start path from clear road.

NEXT STATE denotes the next state to go to. For example if the test distance ahead too small is satisfied, the state distance too small is entered.

## *Showing Code Implements the Diagram Relation*

To show that the program implements this state diagram, it is necessary to show that for each row in the state diagram table:

1. There is code that implements the row.

2. That code is executed whenever the state for the row occurs.

3. Nothing else in the program interferes with the code implementing a row.

Condition 1 follows from the fact that there is a branch in the code for each row in the table. A complete verification would identify the computational path for each row. To illustrate the technique, consider the row with the following values:

state = clear road

condition = current speed < desired speed

actions = increment speed, small delay

next state = start

There is a case statement branch corresponding to the clear road state. Both a small delay and setting the next state to start are executed whenever the clear road branch is entered, using the definition of sequential statement execution in procedural languages. Using the definition of if in procedural languages, increment speed is executed whenever current speed < desired speed.

Condition 3 follows from the following two facts:

1. All code in the case statement implements some row in the table

2. The code for each row is executed only when the conditions for that row are satisfied.

### *Whoops -- A Bug!*

In attempting to verify Condition 2, a bug in the code is found. The code for the exit state is never executed. This is because the condition for exiting the while loop succeeds whenever the state becomes exit. Even worse, the only return statement for the code occurs in the erroneously non-executed code for the exit state. As a result, the code shown here could return undefined values to its calling context, propagating errors up through the program in which it is used.

The solution to this bug is to move the return statement to just below the while statement. Were this done, Condition 2 would be satisfied.

The above bug was not planted, but represents a bug in the code that the authors did not catch before writing this section. The fact that the bug was found while trying to carry out a verification proof illustrates that the proof process exposes bugs by causing the developer to examine code greater detail than when he or she merely inspects the code.

# 7. VV&E for Small Expert Systems

Small expert systems are those for which direct proof of completeness, consistency, and specification satisfaction are practical without partitioning the knowledge base. This chapter discusses techniques for these proofs.

The basic method for verifying properties of small systems is:

1. Represent the property to be verified as a logical formula.

2. Verify the logical formula using one of the following techniques:

- Verify the formula on a case-by-case basis, e.g., by checking each Hoffman Region.
- Apply Boolean algebra simplifications to verify the formula.

## Completeness

To verify completeness the user must demonstrate that for all inputs, the expert system produces *some* conclusion. This is done by:

1. Constructing a logical formula that represents conditions under which the system is complete; this logical formula will be called the *completeness formula.*

2. Showing that the truth value of that formula is TRUE.

If the expert system E is C*artesian,* i.e,. it is required to produce values of more than one variable, then the completeness formula for E is the AND of the completeness formulas for systems which set *each* of the required output variables for E.

For a system E that is required to take one of some set of actions a pseudo-variable is created of which values are the enumerated set of acceptable actions. Then the completeness formula for E is the completeness formula for a system that outputs the value of this variable.

The completeness formula for an expert system in E which sets a single variable v is constructed by an iterative substitution process from an initial formula. That initial formula is:

$$( v = e1) \text{ OR } (v = e2 ) \text{ OR } ... \text{ OR } (v = en) \qquad (7.1)$$

where v = ei is an expression from a rule conclusion that sets v.

It is generally not possible to establish the truth of (7.1) directly. However, the user can build a formula that expresses the truth of (7.1) in terms of the input variables of the system. To build this formula, the user needs the following *hypothesis function* on atomic logical formulas in E:

Let X be a formula of the form,

VARIABLE = VALUE

If there is a rule in E containing X in its conclusion,

H(X) = OR( Hi(X) )

where Hi is the hypothesis (if part) of a rule in E which contains X in its then part.

Otherwise H(X) = X.

Using the function H, one can define a logical formula that expresses (7.1.) in terms of input variables:

Let F0 be a variable over logical formulas.

F0 = (8.1);

while

( F0 contains an atomic formula for which H(X) != X)

F = the result of substituting H(X) for X in f;

return F0;

The resulting logical formula, which will be called COMPLETENESS, expresses completeness in terms of input variables to the expert system E. E is complete if the truth value of this formula is TRUE. To prove that COMPLETENESS is TRUE:

1. Write COMPLETENESS in conjunctive normal form.

2. Eliminate OR's containing logical opposites or all possible values of a variable.

If the resulting logical expression is TRUE, the system is complete. If the resulting logical expression is something else (call it COMPLETE0 for discussion purposes), then COMPLETE0 expresses the conditions under which the system produces a conclusion. Although not logically true, COMPLETE0 may be true because of mathematical theorems or domain knowledge.

Alternatively, NOT COMPLETE0 may be satisfiable. In this case, the expert system E is not complete.

Figure 7.1 below illustrates the above explanation of completeness.

Completeness of Investment Subsystem
To show the completeness of the investment subsystem (call it INV) of KB1, the first step is to construct the formula (7.1) for INV:

investment = stocks OR investment = "bank account" (7.1.a)

Expressing this in terms of input conditions gives

( "Risk tolerance" = high (7.1.b)
AND "Discretionary income exists" = yes )
OR
( "Risk tolerance" = low
OR "Discretionary income exists" = no)

Writing this in conjunctive normal form gives

( "Risk tolerance" = high (7.1.c)
OR "Risk tolerance" = low
OR "Discretionary income exists" = no )
AND
( "Discretionary income exists" = yes
OR "Risk tolerance" = low
OR "Discretionary income exists" = no )

The first term is TRUE because high and low are the *only* possible values for risk tolerance. Likewise the second term is TRUE because yes and no are the only possible values for discretionary income exists. Therefore, the formula expressing completeness of INV is TRUE, and INV is complete.

Figure 7.1: Completeness of Investment Subsystem

## Consistency

To verify consistency, the user must demonstrate that for all inputs, the expert system produces *a consistent set* of conclusions, i.e., that for each set of possible inputs, all the conclusions of the expert system can be true at the same time. (As noted in an earlier chapter, determining which sets of possible conclusions are consistent generally requires expert knowledge.)

To establish consistency, the user must do the following:

1. Construct a logical formula that represents conditions under which consistency fails; this logical formula will be called the *consistency formula.*

2. Show that the truth value of that formula is FALSE.

For a system E that is required to take one of some set of actions, a pseudo-variable is created whose values are the enumerated set of acceptable actions. Then the consistency formula for E is the consistency formula for a system that, perhaps among other things, outputs the value of this variable.

If there are no sets of inconsistent possible outputs, the system is consistent. Some expert systems are designed to recommend a set of components of a solution, and no one component contradicts any other. An investment advisor who recommended to each investor a set of desirable investments would be an example of this. For such systems, consistency is not an issue.

Let I1,...,In be the sets of mutually inconsistent possible conclusions of E. Each I consists of some set of conclusions, e.g.,

$$Ii = \{Ci1, \ldots , Ci(mi)\} \qquad (7.2)$$

where the Cs are possible conclusions of E.

The consistency formula for E is:

$$F(I1) \text{ or } F(I2) \ldots \text{ or } F(In) = \text{FALSE} \qquad (7.3)$$

where

$$F(Ii) = Ci1 \text{ and } \ldots \text{ and } Ci(mi) \qquad (7.4)$$

It is generally not possible to establish the truth of (7.4) directly. A formula can be built, however, that expresses the truth of (7.4) in terms of the input variables of the system. Just as for completeness, this formula is constructed by substituting the OR of rule hypotheses that infer a conclusion for that conclusion. Substituting the hypothesis function (7.2) into (7.4) using the iterative algorithm (7.4) constructs the consistency formula.

The resulting logical formula, which will be called CONSISTENCY, expresses consistency in terms of input variables to the expert system E. E is consistent if the truth value of this formula is TRUE. To prove that CONSISTENCY is TRUE:

1. Write CONSISTENCY in disjunctive normal form.

2. Eliminate ANDs containing logical opposites or other contradictory sets of conjuncts.

If the left hand side of the resulting logical expression is FALSE, the system is consistent. If the resulting logical expression is something else (call it CONSISTENT0 for discussion purposes); then CONSISTENT0 expresses the conditions under which the system produces possibly contradictory conclusions. Although not logically false, CONSISTENT0 may be false because of mathematical theorems or domain knowledge.

Alternatively, CONSISTENT0 may be satisfiable. In this case, the expert system E is not consistent, and is inconsistent for the inputs which satisfy CONSISTENT().

**Consistency of Investment Subsystem:**

To show the consistency of the investment subsystem (call it INV) of KB1, the first step is to construct the formula (7.2.3) for INV. The only set of inconsistent conclusions is:

$$\{\text{investment} = \text{stocks}, \text{investment} = \text{"bank account"}\} \qquad (7.2.a)$$

Therefore, (7.2.3) for INV is:

$$\text{investment} = \text{stocks AND investment} = \text{"bank account"} \qquad (7.2.b)$$

To show INV is consistent, one must show that (7.2.b) is FALSE.
Expressing this in terms of input conditions gives:

( "Risk tolerance" = high (7.2.c)
AND "Discretionary income exists" = yes )
AND
( "Risk tolerance" = low
OR "Discretionary income exists" = no)
= FALSE

Writing this in disjunctive normal form gives:

( "Risk tolerance" = high (7.1.d)
AND "Discretionary income exists" = yes )
AND "Risk tolerance" = low )
OR
( "Risk tolerance" = high
AND "Discretionary income exists" = yes )
AND "Discretionary income exists" = no )
= FALSE

The first term is FALSE because high and low are contradictory values for risk tolerance. Likewise the second term is FALSE because yes and no are contradictory values for discretionary income exists. Therefore, the left hand side of (7.1.d) is an OR of FALSE's, and is FALSE. This establishes the truth of the consistency formula for INV, and therefore INV is consistent.

Figure 7.2: Consistency of I Subsystem

## Specification Satisfaction

While the vast range of possible specifications (as well as the Goedel Incompleteness Theorem makes it impossible to give a general method for proving specifications, there are some particular kinds of specifications where certain methods are useful.

Many valid specifications are not directly provable because they are not expressed in the variables and propositions used for the knowledge base. Before a specification can be proved it must be translated into the variables and relations used in the knowledge base. Translating specifications into the language of a knowledge base requires expert knowledge. Furthermore, this translation process may expose conditions under which the specifications are violated.

**Step 1**: Find all the possible conclusions that are constrained by the specification.

**Step 2**: Show that each of these conclusions are only made when permitted by the specification; i.e. for the specification S, and each conclusion C identified in Step 1,

If C then S(C) (7.5)

where S(C) is the result of substituting the variable values contained in C into S.

Let EC be the conditions under which the expert system E concludes C. EC is computed by procedure (7.5) above. Suppose:

EC implies S(C) (7.6)

Then whenever C occurs, i.e., when EC is true, S(C) is also true. On the other hand, if expert knowledge causes one to question (8.6), there is reason to think that the expert system can conclude C when S(C) is false.

Figure 7.3 shows a reasonable specification for Knowledge Base1.

A reasonable specification for KB1 is that it never recommend an unaffordable investment.
**Step 1**: The conclusion, investment = stock, is an investment that might not be affordable.
**Step 2**: Formulate how each conclusion is affected by the constraint, e.g.,

Expert system concludes "investment = stock" implies stock is affordable.

The successive substitutions of H(X) for X in this statement, using algorithm (7.4), produce a succession of ever more detailed statements about when the specification is true. For INV, these statements are:

If "Risk tolerance" = high
AND "Discretionary income exists" = yes
the stocks are affordable.
If ("Do you buy lottery tickets" = yes (7.3.a)
OR "Do you currently own stocks" = yes)
AND
("Do you own a boat" = yes (7.3.b)
OR "Do you own a luxury car" = yes)
THEN the stocks are affordable.

The truth of these statements depends on expert knowledge. If experts doubt *any* of them, it is probably because the conditions found in KB1 under which it concludes investment = stocks, are not sufficient to guarantee the specifications. In fact, (7.3.a) seems plausible while (7.3.b) seems weak. This indicates that the conditions for concluding the intermediate hypotheses,

If "Risk tolerance" = high
AND "Discretionary income exists" = yes

are not completely expressed in KB1.

Figure 7.3: Example Specification for KB1

### *Specification Based on Domain Subsets*

Many specifications are of the form:

If the input is in some set S, (7.7)

then the output satisfies a logical formula P.

where S is defined by a logical formula C(I) over the input variables, and B(C1,...,Cn)) is a formula built over the conclusions, Ci, of the expert system; i.e. (7.7) becomes:

If C(I) then B(C1,...,Cn) (7.8)

To prove specifications like (7.8), *symbolic evaluation* of the knowledge base is a useful technique; the user can try to prove (7.8) by symbolic evaluation using either forward or backward chaining. With these proof methods, the user simulates the inference engine operating on inputs using the knowledge base. However, instead of actual input values, the only thing known about the inputs is that they satisfy C(I). This may be enough, however, to establish that the if a position of some of the rules are satisfied, then the conclusions will be derived within the said part of the rule. If so, these conclusions have been proved true on the basis of the assumptions, C(I). Further reasoning may lead eventually to showing that B is true.

Here is a forward chaining algorithm to prove B given C(I):

1. Assume C(I) = TRUE. : (7.9)
2. If the truth value of B can be established using known information,

   do so and goto X.
3. If the if part of a previously unsatisfied rule can be satisfied,

   then set the then part of the rule to TRUE and goto 2

   else quit, failing in the attempt to prove B.
4. If B is true,

   then the specification has been proved

   else if B is false

   then the specification is not satisfied.

Here is the backward chaining algorithm:

1. CURRENT = if C(I) then B(C1,...,Cn). (7.10)
2. If the truth value of CURRENT can be established, do so and quit with the following result:

   If CURRENT is true the specification has been proved,

   but if CURRENT is false, the specification is not true.
3. If there is an atomic formula A for which H(A) != A (see 7.2)

   substitute H(A) for A in CURRENT;

   Goto 2.
4. Quit with failure to establish the specification.

Figure 7.4 shows the symbolic evaluation of the KB1 example from chapter 5.

To illustrate symbolic evaluation, the following specification will be verified on the original KB1 of chapter 5:

If current savings < $3000, recommend that the investment is savings account. (7.4.a)

To prove the requirement, one assumes the condition:

"Savings balance" < $3000 (7.4.b)

and tries to prove that the expert system concludes that:

investment = "bank account". (7.4.c)

The strategy for carrying out this proof is to use a modification of the expert system's inference engine. It must be assumed that the inference engine makes inferences according to the rules of propositional logic. It is left to the scheduling strategy programmed into the inference engine to determine which of all the possible inferences that could be made are in fact made. For illustrative purposes, a backward chaining strategy is assumed.

Using a backward chaining strategy to prove that the expert system concludes 7.4.c, the user starts with that conclusion and shows that it is satisfied. The only way to do this in Knowledge Base 1B is to satisfy the "if" part of Rule 2. These conditions are true whenever

"Discretionary income exists" = no. (7.4.d)

Rule 6 makes this conclusion whenever:

"Savings balance" <= $3000. (7.4.e)

so (7.4.a) follows.

Notice that this proof mimics the inference engine of the expert system. In fact, every step of the proof could be carried out by the inference engine except for the last step of concluding (7.4.a). However, a modified inference engine could carry out that step if, whenever a truth value for an inequality was needed, a knowledge base about inequalities was consulted. In fact, such a knowledge base appears in the appendix of this chapter, and contains a rule that says

If X<=C then X<C.

Using this rule, a modified inference engine that consults an knowledge base about when atomic formulas are true is able to *automatically prove the desired condition* (7.4.a) *about the knowledge base*.

Figure 7.4: Symbolic Evaluation

The main differences between actual and symbolic inference engines are:

- Actual inference engines collect actual values for variables and use them in evaluating the conditions of rules to see if those rules fire.
- Symbolic inference engines have available logical conditions about the value of variables, e.g., the hypotheses C(I). Symbolic inference engines use this known information to infer whether atomic formulas in rule hypotheses are true or false. In an appendix to this chapter appear rules for symbolically determining the value of some arithmetic atomic formulas.

To construct a symbolic inference engine from an actual inference engine, the function that determines the truth of atomic formulas is replaced, but leaves the rest of the inference engine code intact. The *actual inference engine* determines the truth of atomic formulas by:

- Looking up the actual values of variables.
- Substituting them into the atomic formula.
- Determining the truth value of the result.

In a symbolic inference engine, the user can also evaluate atomic formulas when only *conditions about* variable values are known, but the actual values are unknown. The symbolic inference engine evaluates atomic formulas by:

- Assuming the known conditions about the variables in the knowledge base.
- Using this information to establish the truth of the atomic formula.

To build a symbolic inference engine requires the user to replace the function for the actual evaluation of atomic formulas with a function for symbolic evaluation, and to leave the rest of the inference alone.

Figure 7.5 list the steps involved for the actual inference engine.

*Actual inference engine*: For KB1, suppose the user said that his or her savings balance was $2000. Then the truth value of the atomic formula:

savings balance < $3000 (7.5.a)

can be determined by substituting in $2000 for "savings balance" to produce:

$2000 < $3000 (7.5.b)

This inequality is seen to be TRUE.

*Symbolic inference engine*: Suppose that:

"savings balance" <= $2000 (7.5.c)

is known to a symbolic inference engine. The symbolic inference engine tries to prove:

"savings balance" <= $2000 (7.5.d)
IMPLIES "savings balance" <= $3000

This formula is seen to be true. In fact, using the following row of the table in the appendix for evaluating atomic formulas,

| ATOMIC FORMULA | TRUTH CONDITION | FALSE CONDITION |
|---|---|---|
| [a,b]=<c | b<=c | a>c |

one may conclude (7.5.d).

Figure 7.5: Symbolic Inference Engine

## Effect of the Inference Engine

The consistency and completeness techniques and the forward and backward symbolic inferences engines presented so far in this chapter are based on the assumption that fairly standard inference engines are used in processing the expert system knowledge base. These standard algorithms are capable of making all the inferences of propositional logic. Inference engines can depart from these algorithms by either error or design. For example, an inference may stop inference after the first knowledge base conclusion.

The most probable effect of departures in the inference algorithm from the standard is that the inference algorithm makes *fewer* inferences than the standard algorithm. An inference engine error of commission, a false inference, is more likely to be found during testing of the inference engine, while an omitted inference is harder to detect.

*Consistency*: Omitted inferences do not affect the above methods for finding consistency. The omissions merely mean that there are fewer conclusions to be inconsistent than expected.

*Completeness*: Omitted inferences *do* affect completeness. The omitted inferences may cause the inference engine not to make an expected conclusion. Where the inference engine is known to omit some propositional logic inferences, it is suggested that completeness be verified using a symbolic version of the same inference engine used on the knowledge base, incomplete though its inferences may be.

*Satisfaction Of Specification*: Specifications can be verified using symbolic versions of the *same* inference engine used to run the knowledge base. This provides the best insurance that the inference engine will actually make inferences that correspond to those made during the proof of the specification according to the rules of logic.

## Inference Engines for Very High Reliability Applications

For applications where very high reliability is required, it is essential that the inference engine be known to make correct inferences. CLIPS (C Language Integrated Production Systems, first released by NASA in 1988) is the only expert system shell claimed to have a certified correct inference engine.

In order to know that an inference engine makes correct inferences, it is necessary that the inference engine be proven correct. One possible standard of correctness is that the inference engine makes all inferences possible with propositional logic given the information gathered from the user and other sources. This, or an alternative standard, should be proved by carrying out a formal correctness proof on the source code of the inference engine, practical only for small, simple ones.

On the basis of this or other proven description of inference engine behavior, algorithms can be constructed like those shown in this chapter for consistency, completeness, and symbolic evaluation for specification satisfaction.

In order to carry out in practice a correctness proof or equivalent description of the inference engine it is necessary that both the source code of the inference engine be available and short and simple enough to allow a proof to be carried out, given the primitive state of program correctness proofs.

# 8. Validating Underlying Knowledge

If there are errors in the knowledge from which a knowledge base is built, there will usually be errors in the performance of the expert system. This chapter discusses methods for validating the knowledge from which a knowledge base is built.

## Introduction

If there are errors in the knowledge from which a knowledge base is built, there will usually be errors in the performance of the expert system. There are several ways that the KB can come to represent incorrect knowledge:

- The expert(s) provide incomplete or incorrect knowledge.
- The knowledge engineer fails to correctly understand or code the expert's knowledge.
- Formalizations of knowledge, e.g. using the range of a variable to test for some underlying condition, may fail to capture all instances of the underlying condition.

There are two kinds of validation that must occur on a knowledge base: *logical* and *semantic*. Logical validation checks how the rules and objects work together to reach logical conclusions. In particular, logical validation checks for *consistency*, i.e., that all the conclusions of the knowledge base can be true at the same time. Logical validation also checks for *completeness*, i.e., that the knowledge base reaches a conclusion for all inputs. While earlier chapters of the Handbook focused on logical completeness and consistency, this chapter addresses semantic correctness and completeness.

Logical completeness and consistency are necessary for a knowledge base to be valid. However, logical completeness and consistency are not sufficient for knowledge base validity. For example, Knowledge Base 1 in the Introduction is logically complete and consistent; it contains no logical errors. However, KB 1 makes investment decisions based only on risk tolerance and discretionary income. It uses no information about actual income, debt, fixed expenses, age and other important inputs to good investment decisions. In other words, while KB 1 is logically correct, it is seriously semantically incomplete. To be valid, a knowledge base must be *semantically complete*, i.e., it must base its decisions on all information considered to be relevant by the expert.

An exception is that thorough testing (see Chapter 10, “Testing”) may show that some information can be left out without affecting performance. However, knowledge that the expert thinks is needed should be included until testing shows that an expert system performs correctly without that knowledge.

Similarly, a knowledge base can be logically consistent but not semantically consistent for its intended application. *Semantic consistency* occurs when all facts, rules, and conclusions of the knowledge base are true for the application for which the expert system is intended. To illustrate the difference between logical and semantic consistency, consider ordinary Euclidean and spherical geometry. Both are logically consistent mathematical systems from which logically consistent expert systems can be built.

However, for everyday life, Euclidean geometry is consistent and spherical geometry is inconsistent with observed facts, while for long distance navigation, the reverse is true.

It is important to note, however, that a knowledge base that is logically inconsistent by definition gives contradictory advice and is therefore semantically incorrect. Likewise, a knowledge base that is logically incomplete fails to provide a solution under some circumstances, and is semantically incomplete. **Logical completeness and consistency are prerequisites for semantic validation of a knowledge base**.

## Validating Knowledge Models

Knowledge models are used as a major component of correctness proofs, but these proofs are worthless if the underlying knowledge about the application domain is false in the domain. Therefore, it is important to validate the knowledge models with domain experts.

There are several ways to validate a knowledge model:

- Use a knowledge model from a standards document in the domain. The standards process that created the model can be assumed to have validated the knowledge in the standard.
- Create a knowledge model through the joint development and consensus of a team of recognized experts in the domain. For example, the knowledge base for Quick Medical Reference, an internal medicine advisor, was created by a series of specialized consensus committees in very specialized fields (e.g., Hepatitis B). The knowledge model created in this way can be assumed to contain the best available expertise, and the participation of multiple experts increases the chances that one of them will catch any error that creeps into their discussions.
- Create a knowledge model with a single expert and review the knowledge model with other experts.

When creating a knowledge model using a single expert where correct performance is critical, it is important to validate this knowledge with outside experts not connected with development of the model. The following steps detail the validation process of the knowledge model:

- Present the knowledge model to the outside experts. In some situations it may be advisable to have someone other than the domain expert, author of the knowledge model, do the presentation, to ensure that professional courtesy does not interfere with critiquing the knowledge model.
- Collect all questions, comments, and objections to the knowledge models, or parts thereof.
- Sort and organize these comments into questions about parts of the knowledge model.
- Organize the questions into a cultural consensus test (see the following sections) to validate individual items.
- Give the test to the outside expert, to determine the extent of agreement on each of the items.
- If some of the items are not validated, perform additional knowledge acquisition and modification of the model to resolve the problems pointed to by the invalidated items. This may include additional discussions, bringing in more experts, literature searches, or redoing parts of the model.

Note that in validating the knowledge model, or in other knowledge validation activities, **it is important to ensure that the specialized expertise of experts used in validation cover the intended domain of the expert system**. Most technical fields today are too big and complex to be mastered in their entirety by a single expert, or even a few experts. Therefore, in critical applications, it is important to validate every part of the knowledge base with experts in that particular specialty. An example of this careful validation was the construction of the Quick Medical Reference expert system for internal medicine, and its predecessor systems Internist and Cadeusius. Although the final system contained nearly a thousand diseases, groups of specialists in particular diseases (e.g. hepatitis B) were brought in to collectively discuss and validate the knowledge base in their particular area of special expertise.

After performing these validation steps it is important to assess the performance of the domain expert (see the later section, Overall Agreement Among Experts). If the current domain expert differs from a consensus of other domain experts, then there are two possible courses of action:

- Replace the domain expert with one who represents a consensus of current domain knowledge.
- Continue the expert system with the disputed knowledge model, with the realization that the system will not reflect a consensus of expert knowledge. In this case it is unlikely that the system will perform in a way that matches a consensus of domain experts. Continuing development is a legitimate course in experimental or non-critical systems but is not advisable in critical expert systems.

An expert system containing knowledge which has not been validated should be used only for applications where there is no serious consequence of an error by the expert system.

## Validating the Semantic Consistency of Underlying Knowledge Items

Even if the expert knowledge has been properly encoded into an expert system knowledge base, the KB will probably produce errors if the underlying expert knowledge is wrong. Therefore, it is important to validate the expert knowledge behind the knowledge base. This is particularly important because there are a number of ways in which errors can creep into the knowledge on which an expert system is built. Some of these errors are:

- The expert is wrong or out of date; in fact, all experts are probably wrong or out of date on a few points.
- The knowledge base was correct when written, but knowledge has changed.
- The knowledge engineer misunderstood the expert.
- Errors were introduced in maintenance.

When a given a fact that has been encoded into the knowledge base, how can one validate that this represents correct expertise? One approach is to do an experiment so that:

- One outcome is expected if the fact represents currently accepted expertise.
- Another outcome is expected if the fact does not represent currently accepted expertise.
- There is a statistical test that discriminates to an acceptable level of confidence between these two cases.

The specialty of cultural consensus within anthropology provides techniques for validating knowledge in a statistically rigorous manner. These techniques can be applied to knowledge validation for knowledge bases as explained below.

The basic method for validating a knowledge item is:

- Ask a panel of experts whether it is true or false.
- Tally the TRUE/FALSE answers.
- Analyze the results statistically.

### *Creating a TRUE/FALSE Test*

In asking the experts to decide if the knowledge item is true or false, it is important not to bias them by letting the expert know which answer agrees with the current assumption in the knowledge base. Do not, for example say, "You agree with this, don't you?". To present the items for validation in a context in which both TRUE and FALSE are a priori equally likely, disregarding the truth of the item(s) being tested, do the following:

1. Start with a collection of TRUE/FALSE questions about half of which are true and half of which are false, and which are about the domain of the knowledge base. It is important that these environment-creating questions are indistinguishable by the test taker from the questions that actually test KB knowledge.

2. Scatter TRUE/FALSE questions that actually test KB items throughout the list of environment-creating questions.

3. Adjust the test if necessary so that TRUE and FALSE have approximately equal probabilities of being right.

Although this method is adequate for the purposes of this handbook, more detailed information about constructing unbiased tests can be found in literature about survey and test design.

## *Giving the Test*

In applying the cultural consensus method to knowledge base validation, there are some issues that must be handled carefully to get maximum information from the test. First of all, the knowledge engineer must realize and explain to the experts that it is not they but the knowledge base that is being tested. The items on the test represent assertions on which the knowledge base is based, and these are being validated by experts. The reason for using multiple experts is not a lack of confidence in any one expert, but a desire to validate assumptions made in the knowledge base to a statistically significant confidence level. It is important to explain this to all the experts used in knowledge base validation to ensure that no hostility toward the knowledge engineer or the project develops. Such hostility that would rob the project of valuable contributions to the knowledge base by the expert.

Secondly, the experts used for validation should be carefully instructed to call an item false if it is not always true. This is to guard against the very real possibility that some of the rules in the knowledge base have entry conditions that are too broad. The test can even be given in a form where there are three answers to each question, TRUE, FALSE and SOMETIMES TRUE. SOMETIMES TRUE and FALSE can be combined as FALSE, i.e., the item was not considered true, when the test is scored.

## *Formulating the Experiment*

Once the test for the knowledge base items has been written, an experiment must be constructed using the test results to validate the items. To do this, the test must be given to a group of experts to evaluate and score the results.

The test must be given to enough experts so that the correctness of each knowledge item based on test results can be distinguished from chance test results. Following is a simple statistical method to validate knowledge base items.

## *Analyzing the Test Results*

A knowledge base item is statistically validated if:

- A majority of the experts answer that the KB item is true (or otherwise supply the test answer(s) that one would predict under the assumption that the experts think the KB item is true).
- The majority is so overwhelming that if the experts did not think the KB item was true, the chance of having results that at least this strongly suggest a belief in the KB item is less than some preassigned threshold, traditionally 5 percent or 1 percent.

Table 8.1 shows the chance of finding unanimous agreement given the "null hypothesis," that the experimental results are due to chance rather than belief in the KB item.

Table 8.1: Confidence Level

| NUMBER OF EXPERTS | CONFIDENCE LEVEL |
|---|---|
| 1 | 50% |
| 2 | 75% |

| | |
|---|---|
| 3 | 87.5% |
| 4 | 94.75% |
| 5 | 96.88% |
| 6 | 98.48% |
| 7 | 99.22% |
| N | 1 - 1 / 2**N |

This means that it is probably a good idea to ask at least four experts to verify each important assumption backing up the knowledge base. When four or more experts agree unanimously, the assumption is reasonably validated. Six to seven experts agreeing provides a high level of confidence in the assumption.

Table 8.2 shows the confidence levels results when one expert disagreeing with the rest of the group:

Table 8.2: Confidence Levels with One Expert Disagreeing

| NUMBER OF EXPERTS | CONFIDENCE LEVEL |
|---|---|
| 1 | 0% |
| 2 | 25% |
| 3 | 50% |
| 4 | 68.75% |
| 5 | 81.25% |
| 6 | 89.06% |
| 7 | 93.75% |
| 8 | 96.48% |
| 9 | 98.05% |
| 10 | 98.93% |
| 11 | 99.41% |
| 12 | 99.68% |

This means that when one expert out of eight disagrees the KB item is validated to a reasonable level and is validated to a high level when one expert out of ten disagrees.

In general, if there are N experts of which M disagree, the confidence level achieved by this level agreement is:

1 - (1 / 2**N) * SUM( m = 0 to M)combinations(M,N)

where combinations(M,N) is the number of combinations of M objects chosen from N.

This is computed by:

combinations(M,N) = M!*(N-M)!/N!

where K! is the factorial of K.

## *Overall Agreement Among Experts*

The above method of validation based on cultural consensus rests on an assumption that the experts share the same basic knowledge, i.e., the same ideas about how to solve the problems covered in the knowledge base, and are validating the specifics of that common approach, as expressed in the knowledge base. Sometimes, however, experts do not agree in their basic knowledge and approach to a class of problems. To detect whether all the experts take the same basic approach to problem solving, observe the following:

1. **Cluster the experts**: Represent each expert as the vector of answers on the TRUE/FALSE test. Find a clustering of the experts based on these vectors.

2. **Test for similarity**: Test to see if all the experts belong to the same cluster.

   2a. **Common cluster**: If all the experts belong to the same cluster, then the computation of item confidence presented above remains valid.

   2b. **More than one cluster**: If there is more than one cluster among the experts, analysis of the differences among experts must be conducted, as discussed below. Then the cultural consistency of individual KB items should be retested.

For the small number of experts that are involved in validating a knowledge base, clusters of experts can be determined by hand inspection of the correlation matrix of test answer similarity of experts.

## *Approaches to Disagreement Among Experts*

When experts do not agree, as evidenced by the existence of more than one cluster of experts, the following approaches are useful:

1. **Throw away outliers**: If it can be determined by interviewing other experts that an expert who is not part of a larger cluster of experts represents a little-held school of thought within their specialty, and if the more mainstream approach represented by the large cluster of experts successfully solves the problems for which the expert system is intended, eliminate the outlier expert from the validation sample of experts.

2. **Choose a valid subset of experts**: If two clusters of experts work from totally different assumptions, pick a cluster that achieves optimal results and use them both as the source of domain expertise and experts for validation. Do not try to include two conflicting schools of expertise in the same knowledge base.

3. **Use the separate approaches as subsystems**: If approaches represented by distinct clusters of experts do better on different subsets of the target domain, it may be possible to build a system where the differing approaches reside in separate expert subsystems. These subsystems could participate in a weighted vote to determine an overall conclusion, where the weight given to a vote is the heuristically determined confidence factor that a particular subsystem can solve the problem

under consideration. Since this approach leads to a more complex, expensive system, it should only be used when the separate approaches are not adequate by themselves.

4. **Analyze disagreements**: Two or more clusters of experts may be a symptom of unresolved controversies within the professional specialty supplying the expertise for the expert system. In this case, the expert system development team needs to decide if there is enough agreement among experts to build an expert system that gives reliable advice in the domain for which it is intended.

## Clues of Incompleteness

Clues that a knowledge base is semantically incomplete may exist within the knowledge base itself. One is that the knowledge base is logically incomplete. Another is that variables, statements, conclusions, etc., are defined but not used. This may indicate that an expert started to supply knowledge that would use them, but never completed that part of the knowledge base. Therefore, the entire knowledge base should be checked for items that are defined but not used, and each one of these should be used or eliminated on expert advice.

## Variable Completeness

Variable completeness is a special case of semantic completeness. A knowledge base is variable complete if it uses all of the important input variables in making its conclusions. The steps in checking variable completeness are:

1. Determine and codify what inputs the KB uses in determining each variable and the truth of conclusions.

2. Ask experts to confirm the knowledge codified in Step 1.

There are two ways to determine and codify the variables used in making decisions:

- Computerized analysis of the knowledge base.
- Keeping careful knowledge acquisition and coding notes.

In either case, the goal is to be able to formulate questions of the form:

- The knowledge base currently uses variables V1,V2,V3...VN to decide X.
    - Are there additional variables that should be in this list of inputs? What are they?
    - Are there input variables that are not needed? If so, then what are they?

Once these questions have been defined they should be presented to experts, possibly first to the experts used in building the expert system, and then to independent experts. The process of asking experts about input completeness should be continued until the variable set stabilizes. Then the variable sets should be validated using the technique described above for knowledge item validation.

## Semantic Rule Completeness and Consistency

Once the inputs to making decisions have been validated, the actual rules that make each decision should be validated. One problem in validating knowledge bases has been that the size of knowledge bases and their relative lack of easily perceived structure makes them difficult for domain experts to read. To lessen this problem, the knowledge base can be partitioned into the pieces that determine the value of each important variable and conclusion. Each such piece represents the knowledge in the knowledge base about a particular subtopic of the domain, and some conclusion drawn from that subtopic. The expert(s) is asked to examine each piece of the knowledge base separately, and answer the following questions:

1. Is the information expressed in the rules that set the value of some particular variable or statement correct?

2. Is the information complete? Or are there other conditions to consider, either in individual rules or as new rules?

By focusing the expert's attention on a single variable at a time and the conditions for setting that variable, a large knowledge base is broken down into pieces that are easier to comprehend.

A backwards chaining strategy can be used to go through the variables and statements in an order that is logical to an expert. Start with the overall outcomes of the knowledge base, and for each pull out all the rules that set that conclusion. Validate these rules. Then do the same for rules that set the conditions in the "if" parts of validated rules. Continue the backward chaining validation process until validated pieces cover the entire knowledge base. The question, "Is this knowledge base piece valid", i.e., is the information correct and complete, can be considered a knowledge item, and validated to the desired level of confidence using cultural consensus, as discussed above. For knowledge bases where reliability is critical, this piecewise validation should be carried out.

## Validating Important Rules

Particular emphasis should be placed on validating rules that cover and appear to cover many inputs or which process critical cases. Rules that appear to cover many cases are those with few atomic formulas in their "if" parts. These rules should be pulled out and validated by experts.

To determine which rules typically handle common cases the knowledge engineer in charge of validation should collect a set of typical input data from one or more experts. Each data set is run on the expert system, keeping track of which rules fired in processing this data. Those rules are presented to the experts for validation.

Exactly the same process is used to validate critical cases; data sets are gathered from experts, the data sets run, and the firing rules validated by the experts.

## Validating Confidence Factors

Rule bases may contain assertions about the confidence of conclusions under various conditions, as illustrated by this rule from PAMEX:

```
if DS = 14
        and NOT Deterioration Cause Indicator = Structural Failure
        and NOT Deterioration Cause Indicator = Weather Severity
        and Skid Number = Low
        and DV2 >= 15
        and DV15 < 30
then conclude Aggregate Spray, confidence = 0.8
and conclude Open Friction Course, confidence = 0.8.
```

A problem in validating the knowledge base is to insure that the confidence values are semantically consistent. In particular, if three rules with many "if" conditions in common have confidence values for a conclusion of, for example, 0.9, 0.85 and 0.5, it is important to insure that the low confidence factor is justified by domain knowledge. Either through a coding error, or because different experts supplied the confidence factors, it is possible that the large difference is an artifact of building the expert system.

The basic strategy for validating the confidence factors is:

- Predict the confidence factors for rule conclusions by estimating them heuristically from the conclusion confidences of similar rules.
- Compare the predicted confidences to those actually written into the knowledge base.
- Validate the confidences where the predicted and actual differ by more than some threshold.

The first step in implementing this validation consists of rule simplification. The following rule simplifications should be carried out before predicting confidence factors:

- From a rule of the form "if A then B and C", form tow rules, "if A then B" and "if A then C", so that the confidence factors of B and C will be validated separately.
- Normalize the relational operators by:
    - Replacing all < and<= operators with > and >= operators.
    - Replacing X>=Y with X>Y OR X=Y.
    - Replacing X!=Y with NOT X=Y.
- Write the “if” parts of rules in disjunctive normal form, i.e., as an OR of ANDs of atomic formulas and negations of atomic formulas.
- From a rule of the form "if A OR B then C" form two rules, "if A then C" and "if B then C", so that the two conditions A and B can be validated separately.

The predicted confidence factors are based only on rules having the same conclusion, i.e., to validate the confidence factor of B in "if A then B", it is only necessary to look at other rules with conclusion B. Therefore, although the rule simplifications multiply the number of rules, partitioning by conclusion breaks the rules into subsets of manageable size.

Confidence factors are assigned to atomic formulas in rules in a two-step process. The first step is to assign confidence factors to the atomic formula itself. The second step is to modify that confidence factor if the atomic formula is the argument of a NOT. If an explicit confidence factor appears with an atomic formula, use that as the initial confidence factor for the formula in a rule. Otherwise, if an atomic formula appears in a rule, use 1 as the initial confidence factor. If an atomic formula does not appear in a rule, use 0.5 as its confidence factor. Now, having defined confidence factors for the atomic formulas themselves, modify them to account for NOT's as follows: if an atomic formula with initial confidence C is an argument of NOT, its confidence is 1-C; otherwise, its confidence is C.

At this point, a confidence factor has been assigned to every atomic formula in every rule "if" part. Given two rules, R1 and R2 for each atomic formula A, let A1, A2 denote the respective confidence factors. Then define:

distance(R1, R2) = sqrt(SUM(atomic formulas A)(A1-A2)**2)))

i.e., the square root of the sum of squares of difference between corresponding confidence factors. Using this distance, an estimated confidence factor can be conducted by using a generalized regression neural network (GRRN), which is described in the appendix to this chapter.

In interpreting the differences between actual and estimated confidence factors, it must be decided how much difference should trigger validation. Small differences of 0.1 and possibly 0.2 probably represent expert judgments. Larger differences may indicate errors in the knowledge base, but may also indicate valid expertise. Confidence factors with large differences between predicted and actual values should be validated in a two-step process. First validate the confidence factors with a single expert, e.g., the project domain expert; secondly, if doubt remains, validate the confidence factors with multiple experts using cultural consensus. Since differences may represent expert knowledge, if the expert validates a confidence factor, it may be accepted as valid, or at least as valid as any other knowledge item supplied by the expert. Like other knowledge items, the single-expert-validated confidence factors may be further validated by multiple experts. However, most of these confidence factor differences reflect the fine structure rather than the major assumptions of knowledge bases, and the priority of validating most of them is small. If the difference is large and the consequence of the difference is judged to be serious, however, the confidence factor should be validated by multiple outside experts.

Given that resources are always limited, it is impossible in practice to validate all the items in a knowledge base. Given the need to triage testing, it is important to note during knowledge acquisition:

- which areas of the knowledge base are the most controversial among experts
- which experts disagree most with their colleagues.

In addition, to select priority items for testing, it is important to perform a hazard analysis of the system containing the expert system. This analysis should extend into the expert system, and define which assumptions in the knowledge base are safety critical.

Given both general areas of disagreement in the knowledge base, and priority areas for safety, the knowledge engineer can set priorities for testing underlying assumptions. It is very important to test

items that are both safety critical and prone to expert disagreement; a system that reasons correctly from false information is likely to fail.

# 9. Testing

This chapter discusses how a simple experiment can be designed to test whether an expert system satisfies a specification.

## Simple Experiments for the Rate of Success

The most common statistic measuring how well a system satisfies a specification is to observe the expected fraction of inputs on which the system will satisfy the specification. One can estimate this fraction of an experiment based on the following steps:

1. Select a data sample.
2. Run the expert system on the data sample.
3. Analyze the experimental data.

## Selecting a Data Sample

Each specification for the expert system is of the form:

> If the input satisfies certain conditions, then the output satisfies certain other conditions.

A sample of N data items for a specification is a set of N data items that satisfies the conditions in the if part of the specification. Furthermore, the sample should satisfy the following additional condition:

> If x is a variable which is thought to affect the reliability of the expert system on the specification, the distribution of x in the sample should approximate the distribution of x in the underlying population.

There are several ways to collect a sample:

**random subsample**: If a sample of data was put aside for testing during the initial phase of the system lifecycle, the experimenter can draw a random subsample from this sample.

**monitoring**: Potential inputs can be collected from the environment where the expert system is to perform. A subset of the observed inputs that satisfy the conditions of the specification to be tested becomes the sample for the experiment.

**generated input data**: Where actual data is not available or practical, a computer program can be used to randomly generate data satisfying the input conditions of the specification.

The size of the sample that should be selected is estimated below.

If a specification has been proved to be satisfied, the existence of the proof may increase the reliability achieved by a test. This effect is also discussed below.

## Estimating a Proportion (Fraction) of a Population

If the expert system is run on N data items, and it satisfies the specification on K of those items, then:

> K/N = the experimental point (i.e., single number) estimate of the proportion of the underlying population satisfying the specification

If the sample size N is sufficiently large, the distribution of sample proportions (the values of K/N) is approximately normal. This occurs when both the following conditions are true:

$$K = N*(K/N) > 5 \qquad (9.1)$$

$$N*(1-K/N) > 5$$

When this is true, the standard error of the proportion is

$$s\_e(K/N) = \text{sqrt}( (K/N)*(1-K/N)/N ) \qquad (9.2)$$

When the conditions (9.1) for normality are not satisfied, the Poisson distribution, discussed below can be used to estimate the satisfaction of a specification.

## The Confidence Interval of a Proportion

In this section the goal is to find an interval of proportions (fractions) of a population since most of the time the observed satisfaction of the specification for a new sample will be in the interval. In particular, the goal is to find an interval such that the probability of the observed satisfaction being in the interval is (sat) for (sat) close to 1.

The steps in computing the interval are:

1. Conduct the experiment to test the specification. Observe:
   - The sample size N.
   - The number of times K the specification was satisfied on the sample.
   - Conduct enough trials so that the requirements for approximate normality are satisfied.
2. Compute s_e(K/N)
3. From a statistical table, find the standard normal deviate (snd) of sat, often called the "z-score" and denoted by z.

The standard normal deviate is a multiple of the standard error marking out a central region of the normal distribution that contains a given fraction of the total area (which is 1) under the normal

distribution. In particular, z(sat) is the number such that the area under the normal distribution between -z(sat) and z(sat) is sat, i.e.,

$$\text{INTEGRAL}_{-z(sat)}^{z(sat)} \text{normal}(x)dx \qquad (9.3)$$

where normal(x) is the standard normal distribution,

$$n(x) = (1/2*pi)* \exp(-x^2/2) \qquad (9.4)$$

While there is no closed form for z, tables of z-scores are widely available in statistics texts; typical values are shown below:

| sat | z(sat) |
|---|---|
| 50% | 0.68 |
| 75% | 1.15 |
| 90% | 1.65 |
| 95% | 1.96 |
| 98% | 2.33 |
| 99% | 2.58 |
| 99.5% | 2.81 |
| 99.8% | 3.08 |

When K successes are observed in N trials, the sat confidence interval is

$$K/N \; +/- \; z(sat)*s\_e(K/N) \qquad (9.5)$$

## Choosing Sample Size

The goal for a system developer is often to show that a system will perform at least as reliably as some threshold. Statistically, this means that with a confidence of at least C, a specification is satisfied in at least fraction F of a sample on which the specification applies. A typical statement of this form is:

> The expert system correctly diagnoses pavement maintenance remedies at least 90 percent of the time with 95 percent confidence.

This means that if another experiment using the same sample size was conducted, at least 95 percent of the time the measured fraction on which the specification is satisfied would be at least 90 percent.

Given a desired fraction F and a confidence level C, the user can obtain the size of sample needed to achieve these parameters in the following way:

1. Conduct a small initial experiment to estimate the fraction on which the specification is satisfied. This initial estimate will be denoted F0. If F0<F and the sample size of the initial experiment guarantees that there is reasonable confidence in F0, the expert system does not satisfy the proportion F. If F0 is equal or only slightly larger than F, the size of the experiment needed to narrow the confidence interval around F0 to exclude F will be unreasonably large; in practice, it will be impossible to statistically validate the satisfaction with proportion F and confidence C.

2. Given that F0 >F, compute:

$$s\_e = (F0-F)/z(C) \qquad (9.6)$$

To achieve F and C, choose a sample size such that the standard error is less than or equal to s_e. This means choosing an N such that:

$$\text{sqrt}( F0*(1-F0) / N ) =< s\_e \qquad (9.7)$$

or

$$N >= F0*(1-F0)/s\_e\text{^}2 \qquad (9.8)$$

For example, if:

preliminary experimental proportion (F0) = 93%

minimum acceptable proportion (F) = 90%

confidence interval = 95%

then

s_e = (93%-90%)/z(95%) = 0.03/1.96 = 0.153

and

N >= 93% * (1-93%) / 0.153^2 = 277.9

This estimate of sample size is approximate, because the preliminary proportion F0 used in the computation is only the result of a small preliminary experiment, and will contain some random error. Therefore, the experimenter should, if possible, design an experiment so that an initial experiment can be continued by testing more data. This is possible provided that the probability of drawing any data item in the continuation of the experiment is the same as drawing that data item in the initial experiment.

## Estimating Very Reliable Systems

For systems that do not fail often it is difficult in practice to observe the five or more failures that causes the proportion to be approximately normally distributed. In this case the Poisson distribution should be used as follows to estimate a confidence interval for the satisfaction proportion.

The Poisson distribution describes the number of occurrences of some random event in given interval of time or region of space. For example, the number of fish over any square meter of a lake, where the lake bottom is uniformly attractive to fish, is approximately Poisson distributed.

The formal requirements for an occurrence to be Poisson distributed include:

- Each occurrence is independent of the others.
- Each interval can potentially contain an infinite number of occurrences.

In practice, the second requirement can be approximated if a large number of occurrences can occur in a region; what is "large" for this purpose will be estimated below.

If the average number of occurrences in a region is L, the probability of finding k occurrences is:

$$P(k) = \exp(-L)*L^k/k! \qquad (9.9)$$

The probability of K or more occurrences is:

$$\text{SUM } \exp(-L)*L^k/k! \qquad (9.10)$$

$$k>=K$$

a series that converges geometrically once $L/k < 1$.

For testing a specification, a region is defined to consist of N trials, where N is a number such that N or more occurrences is very unlikely, as computed by (9.10).

The requirement that a specification is satisfied at a proportion at least F, means that

$$(N\text{-Fail})/N > F \qquad (9.11)$$

where N is the number of trials in a region, and Fail is the number of failures observed in N trials. This means that the number of failures Fail should satisfy is:

$$\text{Fail} < (1\text{-}F)* N \qquad (9.12)$$

This says that the number of failures should be less than the acceptable failure rate, 1-F, times the number of trials in a region. Using (9) the user can compute the probabilities of observing failure rates satisfying (9.12). Denote the sum of these probabilities by

$$P = \text{SUM } P(k) \qquad (9.13)$$

$$k < (1-F)*N$$

Then if P >= F, the expected success rate is at least F.

## How a Proof Increases Reliability

Suppose that in a Hoffman region a specification has been proved and verified in a single experimental trial.

The question to be asked then is:

> What is the probability that the specification would fail on a new trial with inputs in the Hoffman region ?

By the definition of a Hoffman region, all atomic formulas that determine the computational path of the system have the same truth values for the inputs of the second trial. Therefore, the output on the new trial should be identical to that of the first on which the specification was satisfied. The only way for the outcome of the second trial to be different is for a system error to have occurred. Therefore, the probability of a failure on a Hoffman region for which both a proof and a single trial experimental verification is available is the probability of an underlying computer hardware or system software error occurring during the computation. As the Pentium bug illustrates this is a small, but non-zero probability.

In order for a fielded system to perform reliably, the probability of a computer system error must be kept small. However computer system error probability applies approximately equally to all knowledge bases. Therefore, once the underlying reliability of the computer system is established, resources should not be expended testing for this error. In particular, where a proof exists, one experimental trial per Hoffman region is sufficient to verify a specification with probability 1-Fc, where Fc is the probability of a computer system error.

# 10. Field Evaluation, Distribution and Maintenance

Evaluation, which includes field testing, addresses the issue "is the system valuable?" Value is indicated by the degree of end user approval, which in turn determines the extent of acceptance and use of the expert system. Distribution and maintenance of expert systems are addressed in this chapter.

## Field Evaluation

Evaluation is the process of determining the likelihood that once deployed, the expert system will be used whenever appropriate. Evaluation should be an ongoing endeavor to help ensure maximum usage of the deployed system. Pertinent issues in evaluation are:

- Is the system user friendly and do the users accept the system?
- Does the system give "correct" results and is the logic of the system correct?
- Does the expert system offer an improvement over the practices it is intended to supplement?
- Is the system easy to learn and to become proficient on?
- Is the system useful as a training tool?
- Is the system in fact maintainable by other than the developers?
- Can the system be used in the intended work environment?

There are no universally accepted standards for the evaluation of expert systems. In fact it may be quite difficult to achieve. Sometimes evaluation is ignored until very late in system development. However, there are some things which can be done to make the process more effective. First, for systems under development, the developer should design for testing. For completed prototypes this is impossible, however workshops and substantial interaction with the target end users can make the process of field testing much more valuable. It is critical that the end users be kept aware of how important their contributions are and that their efforts are greatly appreciated.

Workshops and follow-up efforts with end users and experts can provide valuable improvements to an expert system and incentives for its use. After the expanded prototype has been constructed based on knowledge from the experts and end users on the development team, a workshop or series of workshops involving a larger community of experts and end users will usually result in major improvements to the system. The participants should be introduced to the computer program (expert system) and to the general concepts used in the development of the system. During the workshops the knowledge structure and the parameters used in the decision making process and their interrelationships should be reviewed. Expected benefits from workshops include:

- A verification check, i. e. review and improvement of the logic used in developing the system rules
- Enhancement of the knowledge base, i. e. finding oversights in the rules and the relationships between them
- More user oriented and user friendly interface
- Development of vested interest in the user community, i. e. establishment of a cadre of field users who want the system to succeed
- A better and more useable system

As part of the evaluation process, the assumptions made during planning of the system should be reexamined. For example during the planning process assumptions on the frequency of use, availability of input data, usefulness of system output, ease of use, etc. should have been documented. These assumptions should all be tested during field trials.

While there may be no universally accepted standards for field evaluation of expert systems, steps similar to those prescribed in Chapter 9, Testing, can improve the process. The big difference between testing and evaluation is that testing focuses on the accuracy of the advice given by the system while evaluation focuses on the degree of user acceptance of the system. The steps suggested for evaluation are as follows:

1. Select evaluation criteria and determine the minimum acceptable performance level for each criterion selected. Ideally these can be found in the requirements document for the expert system. If this information is missing (which is frequently the case) proposed criteria should be provided to the developers and the users of the system. For example:
    - a minium of 90% of the users surveyed will agree that the consistency of answers the system provides offers a marked improvement over past practice and that the quality of answers provided is acceptable.
    - a minimum of 80% of users surveyed will rate the ease of learning and the user friendliness of the system as acceptable or highly acceptable.

- a minimum of 80% of users surveyed will rate the appropriateness of and ease of answering system queries as acceptable or highly acceptable.

A simple to use evaluation form should be prepared depicting these criteria.

2. Specify how each criterion will be measured. For example each of the criteria may be accessed by the users on the following three point scale:

    - highly acceptable
    - acceptable
    - unacceptable

3. Select a sample of users. Ideally a minimum of 12 users should be selected from the population of users. The greater the number of end users in the study and the more representative they are of the target end user community, the more the results can be viewed with confidence.

4. Gather data. Have each user to use the expert system on a minimum of six cases. After each case, have the user fill out the evaluation form, rating each criterion on the three point scale. Present the six cases to the users in random order. Debrief each user by asking for details on the basis of the rating given.

5. Analyze the data. Summarize all of the ratings of each user on each criterion and across all users for each criterion. Determine if the target level of acceptability has been reached or exceeded for each criterion. The data can then be analyzed.

6. Report the results and recommendations. Report on the strengths and weaknesses of the expert system. Concentrate on suggestions for improving the likelihood of user acceptance by emphasizing features which receive low ratings from the most users and those that could be the most quickly and economically improved.

The personnel who distribute the system and provide field support for the field evaluation must be carefully screened and selected. The wrong selection of support personnel can sabotage even the very best of expert systems. The field support personnel must have the following capabilities:

- Expert knowledge of the domain of the expert system. During this phase the field support staff may be called upon to not only provide support for the expert system that is dependent on domain knowledge, but to answer domain specific questions from the end users. A non domain expert can do irreparable damage to the credibility of the system during this phase.

- Sophisticated computer skills. The field support personnel have to go into a strange environment and correctly and efficiently install an expert system and then provide training using this unfamiliar equipment. Installation problems can always be expected.

- Excellent language skills in the language of the domain experts and end users. The field support personnel must be able to express concepts clearly and concisely to the users at their duty station and in their language and to understand the nuances that the end users try to convey. Anything less than FLUENCY in the language of the end users is not acceptable. In addition to language skills the field support personnel must have excellent interpersonal skills.

Some of the activities to be conducted in field testing are:

- Field operating environment - It is necessary to become acquainted with the field operating environment the expert system will be installed in. Even though operating environment was considered through the domain experts and end users, it will in fact appear different to every observer and there will be factors that effect operation that were not considered before the actual installation and field trials begin.

- Installation of expert systems - The expert system will have to be installed on the equipment provided by the end user/tester. This step will usually not be routine as computers or operating systems, etc., may have to be reconfigured to accommodate the expert system.

- Additional training for the end user/tester will need to be provided. Regardless of prior training, the end user/tester will need support to overcome the various nuances that appear during field test conditions. At this point it is also necessary to define the roles of various parties who participate in the field testing. Who runs the expert system? Who approves the use of expert system recommendations? Who applies the recommendations from the expert system? Who actually collects field data? Who does the preliminary screening/preparation of any data collected?

- After the expert system is installed on the end users computer the requirements and terms for the field tests must be reviewed. Competing demands on the end user are always more extensive at the end users duty station than they were during previous meetings. It is critical to get a renewed commitment. The commitment to support the end user/tester in every way possible must also be reaffirmed. The end user must understand how critical his/her support is and that the sponsor values this.

- Specific test cases should be identified and discussed. This includes previously identified sample test cases and new conditions that may be encountered in the field.

- The formal mechanism for incorporating findings from the field tests should be developed and reviewed in detail with the end user/tester. Also the formal mechanism for sharing

information between end users/testers must be developed and discussed with all end users/testers.

A final evaluation of end user satisfaction should take place after testing has demonstrated that the expert system has reached its target scope of coverage and level of accuracy.

## Distributing and Maintaining Expert Systems

Once the expert system development effort has been completed, the tasks of distribution and maintenance begin. Although there are no fixed rules governing these tasks, there are some general guidelines which can make these tasks easier to perform.

### *Distribution*

There are three major criteria that a developer should follow in order to facilitate the distribution of a given expert system.

1. Identify and involve the user community before starting the development of an expert system. This should insure that the expert system actually solves a real set of problems. Also it will give the user community a vested interest in the testing and application of the system.

2. Develop the system using standard hardware and software. Although there are a number of exotic pieces of expert systems hardware and software, the cost of these items is often high and the uncertainty of survival of these products in the market place makes it unreasonable to expect potential users to procure them just to use an expert system.

3. Use development software that does not require distribution licenses or where an unlimited distribution license can be purchased for a reasonable fee. The time and funding expended on paying fees for each system distributed, as required for many available development tools, can become an unwanted administrative and financial burden.

### *Maintenance*

The task of system maintenance is one that should be planned for from the inception of the expert system development project. Maintenance can be facilitated by following a few good development rules. These include:

1. Design the expert system to be as transparent as possible. Since the system maintenance will probably not be conducted by system designers, it is necessary that the structure of the expert system be as straightforward and clear as possible.

2. The developers should use logical and understandable names for objects and knowledge structures within the system and avoid the use of cryptic names and obscure abbreviations.

3. The developers should also avoid the use of overly complex and obscure software structures, even though their use may provide some type of performance benefits.

4. Simplicity should be one of the guiding principals in the development effort.

The system must be well documented. The documentation should be produced as the system is developed, not as an after thought after the system is finished. The knowledge engineer should:

- Identify where the system's knowledge resides (e.g., in the knowledge base in the form of facts and rules and in the inference engine in the form of heuristic search techniques).
- Document the inference procedures that the system uses in producing solutions.
- Ensure that as part of the documentation an explicit model of the problem solver is included
- Ensure that the documentation also provides a comprehensive and well documented test procedure for the system

The expert system itself should contain an extensive set of both user "help" text and explanation text which explains how the system produced a given solution.

The documentation and the help and explanation text should be produced during the development phase and not added after the system has been built. One of the guiding principles that developers should use is "a poorly documented system will have a short useful life."

Each version of a given expert system should have a version number. This will make it easier to provide users with up-dated copies of the system.

Establish a mechanism for soliciting, receiving and acting upon feedback from the user community. This will facilitate the identification and removal of "bugs" in the system and will also make it easier to retrofit the system to satisfy specific user community needs after the system has been distributed to the user community.

# Appendices

## 1. Symbolic Evaluation of Atomic Formulas

A common type of atomic formula in a rule-based expert system is of the form:

<VARIABLE> <RELATION> <CONSTANT> (A.2.1)

where <RELATION> is one of the relations, =, <, >, <=, or >=.

The following table shows when an atomic formula of this form is true or false given conditions on <VARIABLE> of same form, A.2.1.

In this table, "TRUTH CONDITION" specifies conditions under which the atomic formula is true for all numbers in the interval. "FALSE CONDITION" specifies conditions under which the atomic formulas are false for all numbers in the interval. The following restrictions on the variables a, b and c apply:

a is in [-INFINITY,INFINITY)
b is in (-INFINITY,INFINITY]
c is in (-INFINITY,INFINITY)

| ATOMIC FORMULA | TRUTH CONDITION | FALSE CONDITION |
|---|---|---|
| (a,b)<c | b<=c | a>=c |
| [a,b)<c | b<=c | a>c |
| (a,b]<c | b<c | a>=c |
| [a,b]<c | b<c | a>=c |
| | | |
| (a,b)=<c | b<=c | a>=c |
| [a,b)=<c | b<=c | a>c |
| (a,b]=<c | b<=c | a>=c |
| [a,b]=<c | b<=c | a>c |
| | | |
| [a,b]=c | a=b=c | a != b or a != c or b != c |
| | | |
| (a,b)>c | a>=c | b<=c |
| (a,b]>c | a>=c | b<c |
| [a,b)>c | a>c | b<=c |
| [a,b]>c | a>c | b<c |
| | | |
| (a,b)=>c | a>=c | b<=c |
| [a,b)=>c | a>=c | b<=c |
| (a,b]=>c | a>=c | b<c |
| [a,b]=>c | a>=c | b<c |

## 2. General Regression Neural Nets

A general regression neural net (GRNN) is a method for estimating a function from a set of its values at particular points in its domain. Although the GRNN algorithm can be put in the form of a neural net, it is best understood as an interpolation. In particular, GRNN interpolates from known data points by computing a weighted average of nearby points. The weights in this average decay exponentially with distance from the point where the function is being estimated.

**Notation**

The following notation will be used:

- Uppercase letters (e.g., P, X, X2 etc) denote points in the input space.
- Lowercase letters with subscripts represent numbers for different fields (axes) for the point named by the corresponding uppercase letter. The subscripts identify which axis the number represents. Axis subscripts follow any subscripts that are part of the name of the point. Examples: p2, xi, x2i.

**Prerequisites for GRNN**

To carry out a GRNN computation, it is necessary that a distance function be defined between any two points in the input domain. The Euclidean distance function works well for GRNNs, although any distance function can be used. The Euclidean distance is defined by:

d(P1, P2) = sqrt( SUM( over fields i)(p1i - p2i)**2)))

A weight function from pairs of points to real numbers is defined as follows:

w(P1,P2) = exp(-K*d(P1,P2))

In other words, the weight assigned to P2 for a GRNN estimate at P1 decays exponentially with the distance from P1 to P2. K is parameter that determines how fast the decay occurs.

**The GRNN Interpolation**

Following is the GRNN interpolation of a function fn:

grnn(P1) = SUM(all points P2 in data set)w(P1,P2)*fn(P2))

This says that the GRNN estimate of *fn* at a point is the weighted average of the known function values, where the weights decay exponentially with distance from the point where the estimate is being made.

## 3. Verification and Validation: Past Practices

Significants numbers of articles on verification and validation of knowledge-based systems first appeared in the literature in the early 1980's. Many authors who have written about or

attempted the verification and/or validation of knowledge-base systems have their own definition of the concepts. The method that they use or the system that they design to accomplish the task(s) is usually a reflection of that particular definition. A few authors have asserted that verification and validation are the same.

The following tables summarize past work in verification and validation. Complete references appear in the bibliography.

VALIDATION METHODS THAT HAVE BEEN USED:

Table A.3-1: Validation Methods

| METHOD | EXPERT SYSTEM | REFERENCE |
|---|---|---|
| Turing Test Variation | Mycin<br>KBSCD | Yu, et al., 1979<br>Agarwal, Kannan,Tanniru,1993 |
| Simple Comparison with Expert | Diabetes Mellitus<br>Tegument<br>Hemody. Monitoring | Lehmann, et. al., 1993<br>Potter & Ronan, 1987<br>Koski, et. al., 1991/92 |
| Comparison w/Expert Using Sensitivity Analysis | ESPE (Tool Set)<br>Prospector | Franklin, et. al., 1988<br>Gaschnig, 1979 |
| Comparison w/Expert Using Freq. Anal. & Distance Anal. | PNEUMON - IA | Verdaguer, et. al., 1992 |

VERIFICATION METHODS THAT HAVE BEEN USED:

Table A.3-2: Verification Methods

| METHOD | TOOL (If Exists) | REFERENCE |
|---|---|---|
| Tables & Pairwise Rule Comparisons | Rule-Checker Check | Suwa, Scott, Shortliffe, 1982<br>Nguyen, et al.,1987 |
| Decision Tables of 'Contexts' | ESC<br>GRAFCET | Cragun & Steudel, 1987<br>Renard, Sterling, Brosilow, 1993 |
| Meta-Knowledge | EVA<br>Valid | Stachowitz, Combs, 1987<br>Laurent (ESPIRIT-II) |
| Analytical Hierarchy Process | | Bahill, Jafar, Moller, 1987 |
| **Graphs:**<br>Constraint Connection<br>Flowgraph<br>Parameter Dependency Network | | <br>Freeman, 1985<br>Fenton, Kaposi, 1987<br>Agarwal, Tanniru, 1992 |
| Petri - Nets | | Agarwal & Tanniru, 1992<br>Liu & Dillon, 1991 |
| **Partitioning:**<br>Graph-Based<br>Clustering<br>Clustering Algorithm<br>Category Partition Method Testing | | <br>Jacob & Froscher, 1986<br>Cheng & Fu, 1985<br>Jacob & Froscher, 1990<br>Amla & Ammann, 1992 |
| Incidence Matrix Technique | IMVER | Coenen, Bench-Capon, Kent, 1994 |
| Ripple-Down Rules | | Kang, Gambetta, Compton, 1994 |

DOMAIN - INDEPENDENT SOFTWARE TOOLS USED FOR V. & V.:

Table A.3-3: V&V Software

| TOOL | PURPOSE | METHOD USED | REFERENCE |
|---|---|---|---|
| RITCaG | Validation | Test Case Generator | Gupta, Biegel, 1990 |
| un-named | Validation | Runs Test Cases | Kang & Bahill, 1990 |
| ESPE | Validation | Sensitivity Analysis | Franklin, et al., 1988 |
| Check | Verification | Tables | Nguyen et al., 1987 |
| ESC | Verification | Decision Tables | Cragun, Steudel, 1987 |
| GRAFCET | Verification | Graphical Design Lang./Dec. Tables | Renard, Sterling, Brosilow, 1993 |
| un-named | Verification | Decision Tables | Vanthienen,Dries, 1993 |
| EVA | Verification | Meta-language | Stachowitz,Combs,1987 |
| Valid | Verification | Meta-language | Jean-Pierre Laurent (ESPIRIT-II project) - Europe |
| BEACON | Verification | Graphs | Freeman, 1985 |
| un-named | Verification | Layered Support Graphs | Valiente, 1992 |
| VALIDATOR | Ver. & Valid. | Syntax & Semantics Checks | Jafar & Bahill |
| COVER | Verification | First-Order Logic | Preece, et al. 1992 |
| Expert Choice | Verification | Analytical-Hierarchy Process | Bahill,Jafar, Moller, 1987 |
| Spot | Verification | Prolog Rule Base | Lane, 1989 |
| KB-Reducer | Verification | KB reduction | Ginsberg, 1988 |
| IMVER | Verification | Incidence matrices | Coenen, Bench-Capon, Kent, 1994 |
| un-named | Verification | Clustering Algorithm | Jabob & Froscher, 1990 |
| in-progress | Ver. & Valid. | Meta-language, GUI, Visual Guide to Rule in Flow-Graphs | Traylor, Schwuttke, Quan, 1994 (JPL-NASA) |

# 4. Knowledge Base 1 Illustrations

## ILLUSTRATIONS OF KNOWLEDGE BASE 1

The knowledge base 1 (KB1) has six rules. There are seven variables which can take two possible values. It is, therefore, a seven dimensional, binary problem. Let's focus on rule 3 to understand the illustrations of KB1. It has two hypotheses, and one conclusion. The hypotheses are "Do you buy lottery tickets?"="yes", and "Do you currently own stock?"="yes". They are associated with the logical operator "or". The consequent is "Risk Tolerance"="low".

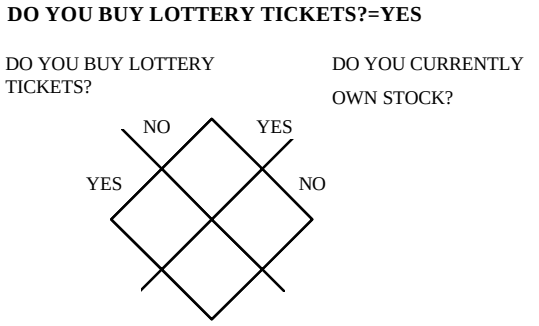



**Figure 1**

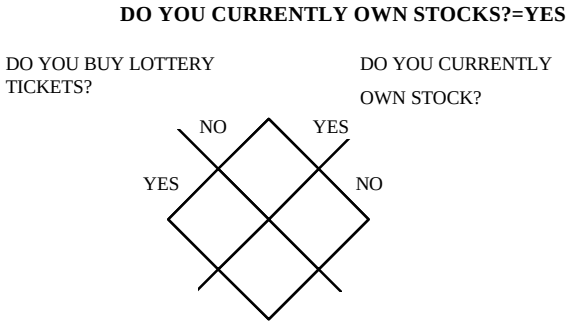



**Figure 2**

For the two variables of the hypotheses in rule 3, there are two possible values: "yes" or "no". The number of possible combinations of values for the variables is four. These four combinations appear in figure 1 as four square regions defined by the closed boundary (defining the domain of the variables) and the line boundaries separating the possible values for each variable. Each square is a Hoffman region.

If variable "Do you buy lottery tickets? " is assigned a value "yes", then two of the four regions are relevant. In figure 1, they are shown with a hatch. The two regions corresponding to hypothesis "Do you currently own stock?"="yes" are hatched in figure 2.

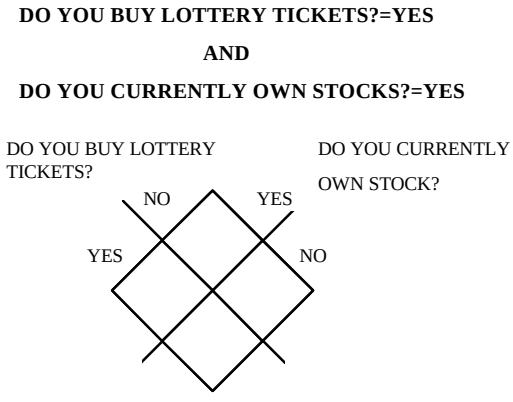



**Figure 3**

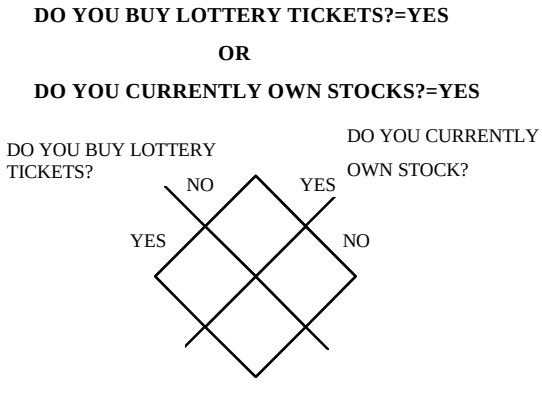



**Figure 4**

In two dimensions, a Hoffman region is a surface as shown in this example. In three dimensions, it would be a volume, ect...

The logical operators are “and”, “or”, and “not”. The last one is obvious in the case of a binary system: “not””yes”=“no”. In figure 1 and 2, the Hoffman regions corresponding to each hypothesis of rule 3 are hatched. When combined with an “and” logical operator, the intersection of the two sets of Hoffman regions that logical expression. It is shown in figure 3. The intersection in this case is a unique Hoffman region.

In rule 3, an “or” logical operator connects the two hypotheses. In this case, the union of the two sets of Hoffman regions is taken , as shown in figure 4.

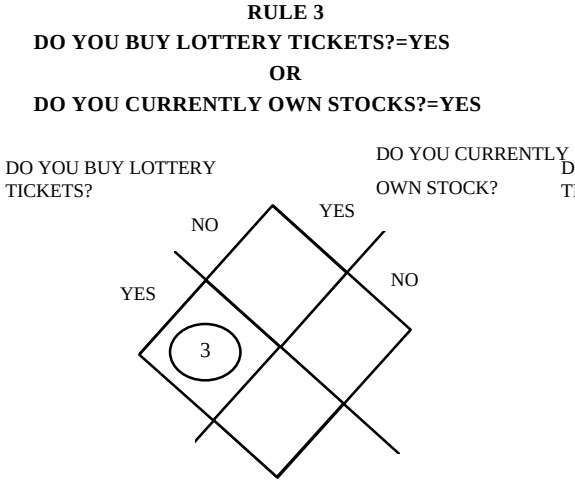



**Figure 5**

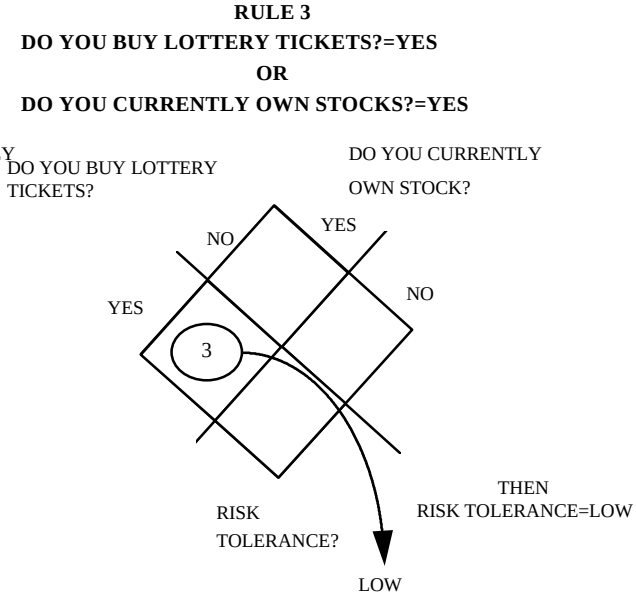



**Figure 6**

Next, the region defined by the logical expression of hypotheses is labeled with its rule number. For rule 3, the three Hoffman regions are labeled with a circled 3, as shown in figure 5. The consequent for the rule is linked to the label of the region of hypotheses. In figure 6, a curved arrow starts at the circled 3, and ends at the value “low” of the variable “Risk Tolerance”.

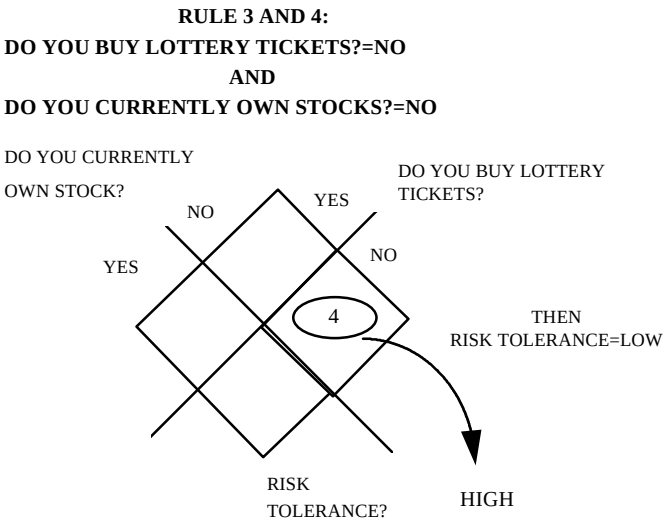



**Figure 7**

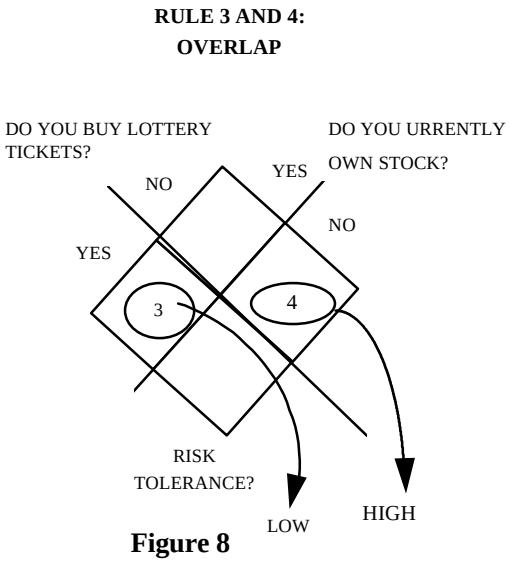



**Figure 8**

At this point, each rule can be represented using this scheme. Rule 4 has the same variables in its hypotheses and conclusions. Figure 7 shows the graphical representation of rule 4, and figure 8 shows rules 3 and 4 together.

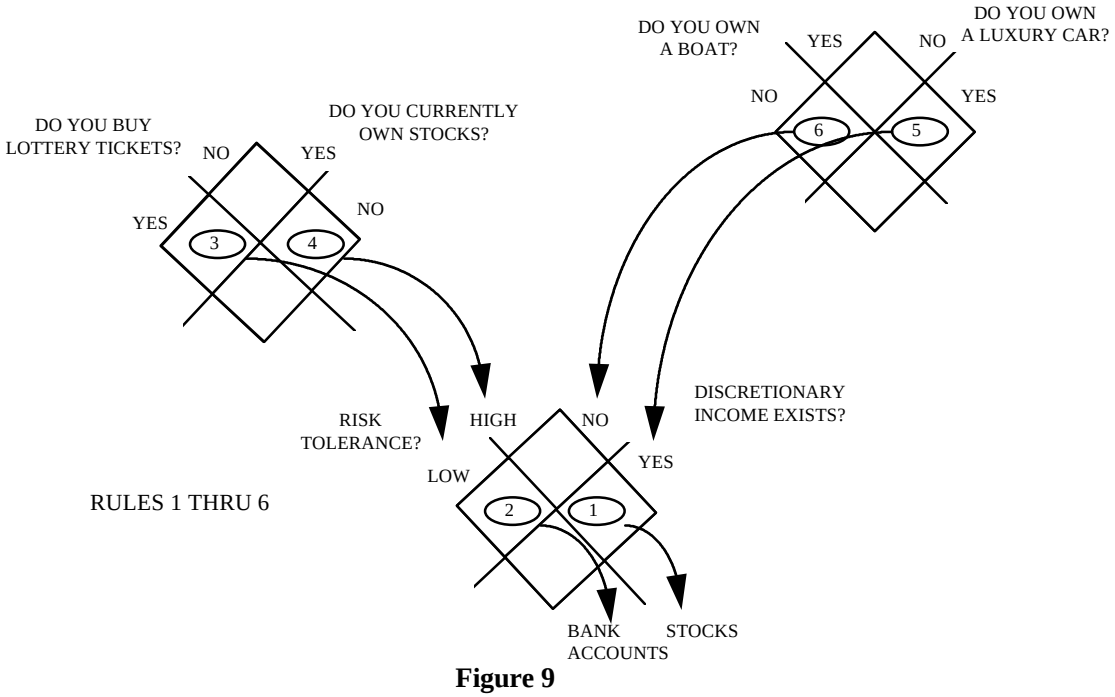



**Figure 9**

All six rules are shown in figure 9. Note that three clusters of rules become apparent: {R3, R4} in the upper left corner, {R5, R6} in the upper right corner, and {R1, R2} in the lower center of the figure..

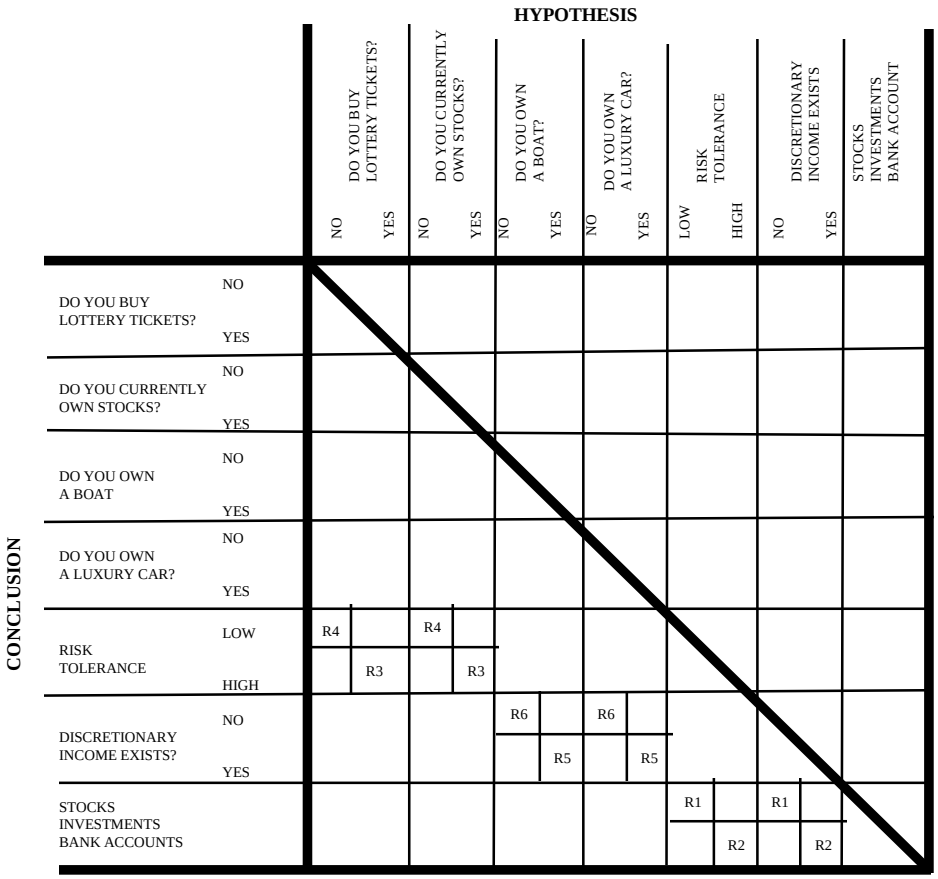
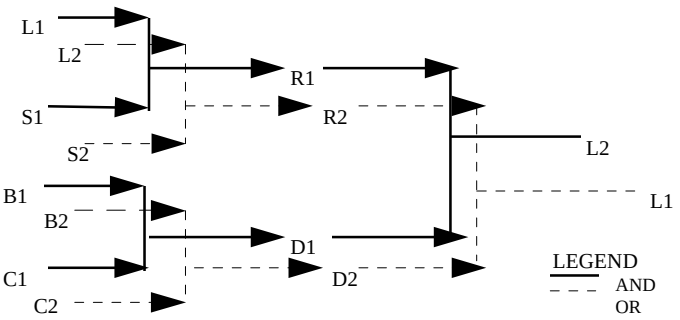



**Figure 10**

For knowledge bases other than binary systems and with more than two hypotheses in rules, an alternative illustration is proposed. An incidence matrix, with rule numbers as values, is developed. The rules are clustered using their commonality of hypotheses and conclusions. The clusters are then ordered so that the bandwidth of the incidence matrix is minimum. Within a cluster, the hypotheses are placed before the conclusions. Figure 10 shows the final incidence matrix for KB1. Note that the partitions are evident. There are three sub-matrices found in the lower triangle of the incidence matrix. They are the smallest matrices which include all variables of a cluster.

| | | |
|---|---|---|
| DO YOU BUY LOTTERY TICKETS? | NO | L1 |
| | YES | L2 |
| DO YOU CURRENTLY OWN STOCKS? | NO | S1 |
| | YES | S2 |
| DO YOU OWN A BOAT? | NO | B1 |
| | YES | B2 |
| DO YOU OWN A LUXURY CAR? | NO | C1 |
| | YES | C2 |
| RISK TOLERANCE? | LOW | R1 |
| | HIGH | R2 |
| DISCRETIONARY INCOME EXISTS? | NO | D1 |
| | YES | D2 |
| INVESTMENTS | STOCKS | I1 |
| | BANK ACCOUNT | I2 |



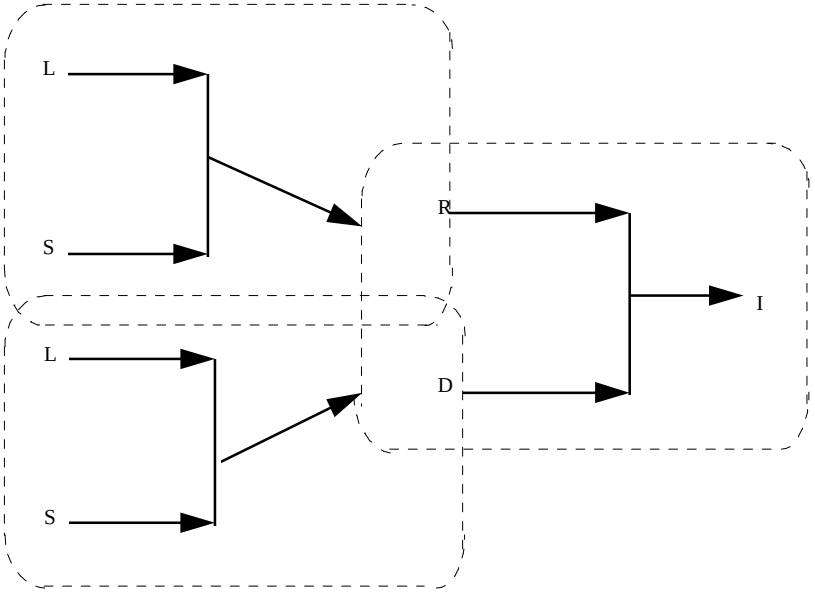




**Figure 11**

Another method of representing a knowledge base is the petri-net method. Each variable is given a name, and each value, a digit. For example, the variable “Do you buy lottery tickets?” is assigned the letter “L”, and the values “no” and “yes”, “1” and “2”, respectively. For example, the hypotheses “Do you buy lottery tickets?”=“no” is assigned to variable “L1”. In Figure 11, a table in the upper left corner lists the correspondence between the hypotheses and the variables for the knowledge base KB1. There are also two graphical representations of KB1. The upper one relates the variables without details of the logical syntax. The lower one provides those details. The dashed line indicates that the hypotheses are subjected to logical operator “or”, and a solid line, “and”, as shown in the legend.

## CASE STUDIES OF COMPLETENESS AND CONSISTENCY

The partition {R3,R4} of KB1 is used to illustrate the concept of completeness and consistency. In cases other than the first one, the two rules are modified by changing either the logical operator or the conclusions.

**CASE 1:** Complete and consistent subsystem

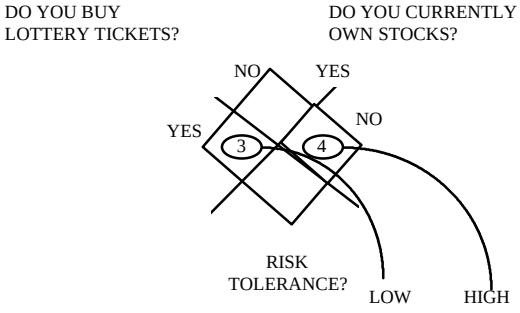



**FIGURE 1**

RULE 3:
DO YOU BUY LOTTERY TICKETS?=YES
OR
DO YOU CURRENTLY OWN STOCKS?=YES
THEN
RISK TOLERANCE=LOW

RULE 4:
DO YOU BUY LOTTERY TICKETS?=NO
AND
DO YOU CURRENTLY OWN STOCKS?=NO
THEN
RISK TOLERANCE=HIGH

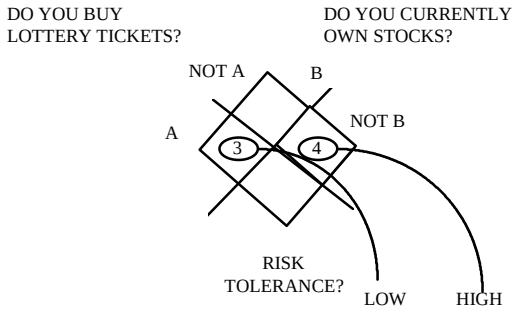



**FIGURE 2**

COMPLETENESS CHECK FOR SUBSYSTEM
LOW OR HIGH
(A OR B) OR (NOT A AND NOT B)
(A OR B OR NOT A) AND (A OR B OR NOT B)
TRUE AND TRUE
TRUE
COMPLETE

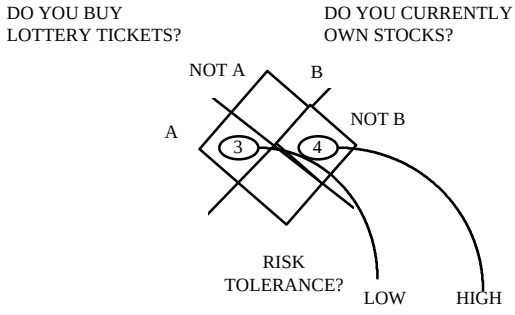



**FIGURE 3**

CONSISTENCY CHECK FOR SUBSYSTEM
LOW AND HIGH
(A OR B) AND (NOT A AND NOT B)
(A AND NOT A AND NOT B) OR (B AND NOT A AND NOT B)
FALSE OR FALSE
FALSE
CONSISTENT

All Hoffman regions are assigned to a unique rule. The results of the formal procedure for checking completeness and consistency are shown in figures 2 and 3. In both checks, the procedures starts at the conclusions. A logical expression is built-up with all possible values of the variables in the conclusions.

**CASE 2:** Incomplete but consistent partition

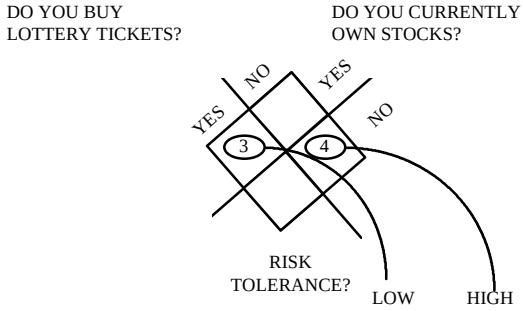



**FIGURE 4**

RULE 3:
DO YOU BUY LOTTERY TICKETS?=YES
AND
DO YOU CURRENTLY OWN STOCKS?=YES
THEN
RISK TOLERANCE=LOW

RULE 4:
DO YOU BUY LOTTERY TICKETS?=NO
AND
DO YOU CURRENTLY OWN STOCKS?=NO
THEN
RISK TOLERANCE=HIGH

The logical operator in rule 3 was changed from an "or" to an "and". Two Hoffman regions are without rule assignment shown by blank patterns. This partition has an incomplete set of rules.

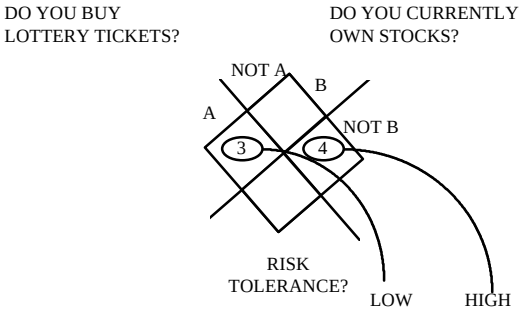



**FIGURE 5**

COMPLETENESS CHECK FOR SUBSYSTEM
LOW OR HIGH
(A AND B) OR (NOT A AND NOT B)
(A AND B OR NOT A) AND (A AND B OR NOT B)
TRUE AND TRUE
TRUE
COMPLETE

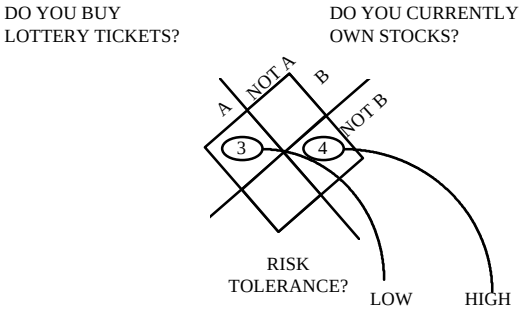
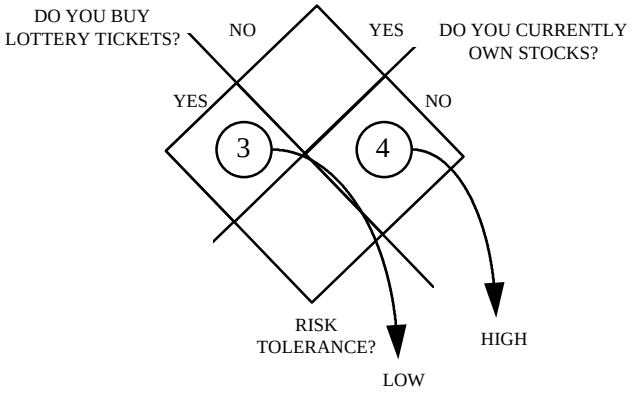



**FIGURE 6**

CONSISTENCY CHECK FOR SUBSYSTEM
LOW AND HIGH
(A AND B) AND (NOT A AND NOT B)
FALSE
CONSISTENT

**CASE 3:** Complete but inconsistent partition

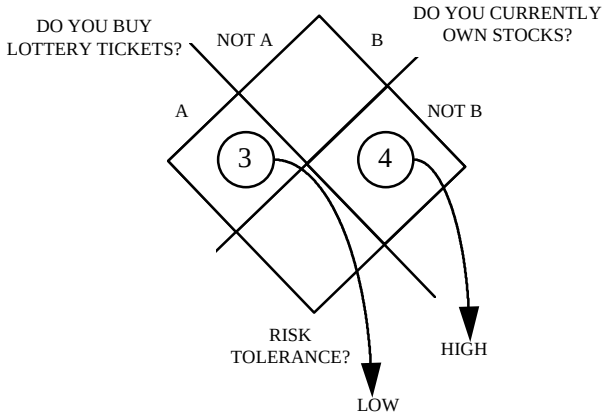



RULE 3:
DO YOU BUY LOTTERY TICKETS?=YES
OR
DO YOU CURRENTLY OWN STOCKS?=YES
THEN
RISK TOLERANCE=LOW

RULE 4:
DO YOU BUY LOTTERY
TICKETS?=NO
OR
DO YOU CURRENTLY OWN
STOCKS?=NO
THEN
RISK TOLERANCE=HIGH

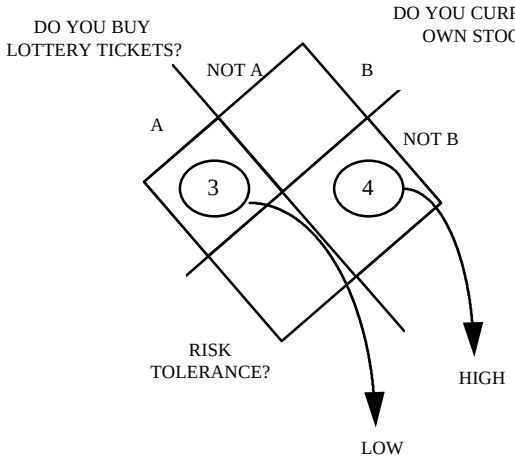
**Figure 7**

The logical operator in rule 4 was changed from an “and” to an “or”. Two Hoffman regions are assigned to two distinct rules shown here by an overlap in the hatch patterns. The partition has an inconsistent set of rules.




COMPLETENESS CHECK FOR SUBSYSTEM
LOW OR HIGH
(A AND B) OR (NOT A AND NOT B)
TRUE
COMPLETE

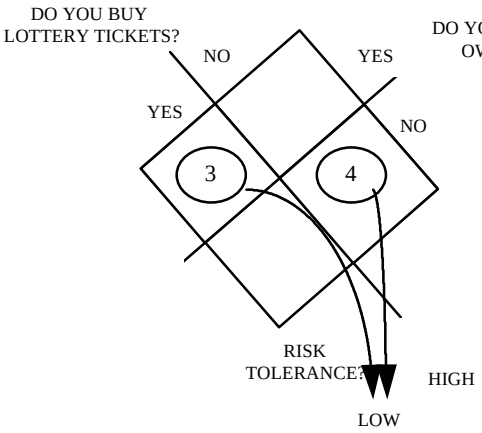
**Figure 8**




CONSISTENCY CHECK FOR SUBSYSTEM
LOW AND HIGH
(A OR B) AND (NOT A OR NOT B)
(A OR B AND NOT A) OR (A OR B AND NOT B)
TRUE OR TRUE
TRUE
INCONSISTENT

**Figure 9**

**CASE 4:** Rules that can be lumped




RULE 3:
DO YOU BUY LOTTERY TICKETS?=YES
OR
DO YOU CURRENTLY OWN STOCKS?=YES
THEN
RISK TOLERANCE=LOW

RULE 4:
DO YOU BUY LOTTERY TICKETS?=NO
AND
DO YOU CURRENTLY OWN STOCKS?=NO
THEN
RISK TOLERANCE=LOW

**Figure 10**

The consequent in rule 4 was changed to be the same as the one for rule 3. The two rules are consistent, but they should be lumped into one.

## ILLUSTRATIONS FOR PARTITIONING OF KNOWLEDGE BASE 1

The concept of relations was introduced in chapter 7. It is applied to KB1 to determine its subsystems. Each subsystem can be represented by a function which has a domain and a range shown in figure 1 for the subsystem "Risk Tolerance ".

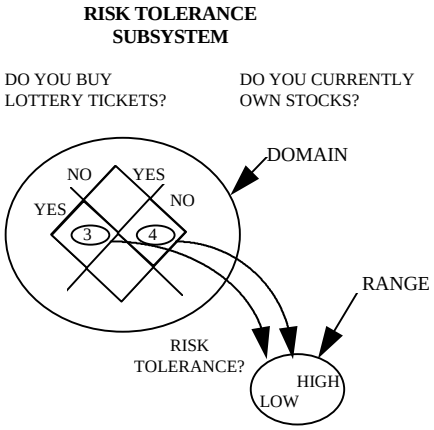



**Figure 1**

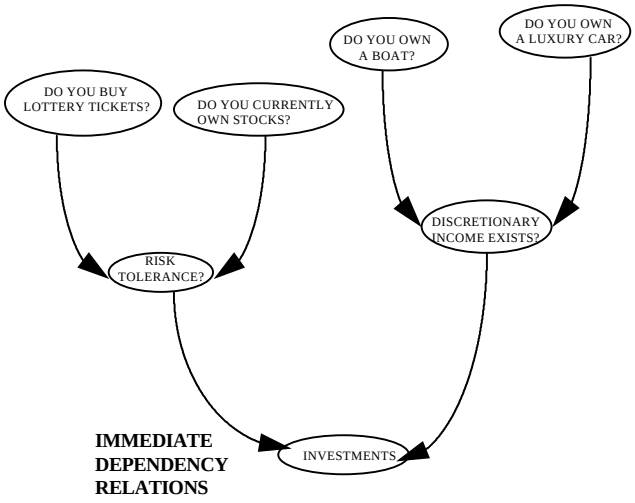



**Figure 2**

In figure 2, the immediate dependency relations of variables on variables are shown by connections. In order to identify the clusters of variables of each subsystem, an algebric procedure was defined. First, two relations are input: 1) the immediate dependency of rules on variables (shown in figure 3), and 2) the immediate dependency of variables on rules shown in figure 4).

| DEPENDENCY RELATIONS OF RULES ON VARIABLES | RULE 1 | RULE 2 | RULE 3 | RULE 4 | RULE 5 | RULE 6 |
|---|---|---|---|---|---|---|
| DO YOU BUY LOTTERY TICKETS? | | | | | 1 | 1 |
| DO YOU CURRENTLY OWN STOCKS? | | | | | 1 | 1 |
| DO YOU OWN A BOAT | | | 1 | 1 | | |
| DO YOU OWN A LUXURY CAR? | | | 1 | 1 | | |
| RISK TOLERANCE | 1 | 1 | | | | |
| DISCRETIONARY INCOME EXISTS? | 1 | 1 | | | | |
| INVESTMENTS | | | | | | |

VARIABLES IN HYPOTHESES

RULE

**Figure 3**

| DEPENDENCY RELATIONS OF RULES ON VARIABLES | DO YOU BUY LOTTERY TICKETS? | DO YOU CURRENTLY OWN STOCKS? | DO YOU OWN A BOAT? | DO YOU OWN A LUXURY CAR? | RISK TOLERANCE? | DISCRETIONARY INCOME EXISTS? | INVESTMENTS |
|---|---|---|---|---|---|---|---|
| RULE 1 | | | | | | | 1 |
| RULE 2 | | | | | | | 1 |
| RULE 3 | | | | | | 1 | |
| RULE 4 | | | | | | 1 | |
| RULE 5 | | | | | 1 | | |
| RULE 6 | | | | | 1 | | |

VARIABLES IN CONCLUSIONS

RULE

**Figure 4**

VARIABLES IN CONCLUSIONS

| IMMEDIATE DEPENDENCY RELATIONS OF VARIABLES ON VARIABLES | DO YOU BUY LOTTERY TICKETS? | DO YOU CURRENTLY OWN STOCKS? | DO YOU OWN A BOAT | DO YOU OWN A LUXURY CAR? | RISK TOLERANCE | DISCRETIONARY INCOME EXISTS? | INVESTMENTS |
|---|---|---|---|---|---|---|---|
| VARIABLES IN HYPOTHESES: DO YOU BUY LOTTERY TICKETS? | | | | | 2 | | |
| DO YOU CURRENTLY OWN STOCKS? | | | | | 2 | | |
| DO YOU OWN A BOAT | | | | | | 2 | |
| DO YOU OWN A LUXURY CAR? | | | | | | 2 | |
| RISK TOLERANCE | | | | | | | 2 |
| DISCRETIONARY INCOME EXISTS? | | | | | | | 2 |
| INVESTMENTS | | | | | | | |

**Figure 5**

The terms left blank in the matrix are zero. The product of matrix A from figure 3 and matrix B from figure 4 is called matrix C, shown in figure 5. If subjected to a boolean operation, its non-zero terms become unity. It corresponds to all connections in figure 2.

In figure 5, the immediate dependency relation matrix is shown prior to the boolean operation. The composition of this relation may be btained by multiplying matrix C by itself. Figure 6 shows the result as matrix D. It corresponds to the connection shown in solid line in figure 7, remembering that the composition operation provides all possible paths to an output from the inputs.

The dependency relation is the union of the immediate dependency relation and the composition operation. It is shown in figure 8 and 9.

VARIABLES IN CONCLUSIONS

| COMPLETION | DO YOU BUY LOTTERY TICKETS? | DO YOU CURRENTLY OWN STOCKS? | DO YOU OWN A BOAT | DO YOU OWN A LUXURY CAR? | RISK TOLERANCE | DISCRETIONARY INCOME EXISTS? | INVESTMENTS |
|---|---|---|---|---|---|---|---|
| VARIABLES IN HYPOTHESES: DO YOU BUY LOTTERY TICKETS? | | | | | | | 1 |
| DO YOU CURRENTLY OWN STOCKS? | | | | | | | 1 |
| DO YOU OWN A BOAT | | | | | | | 1 |
| DO YOU OWN A LUXURY CAR? | | | | | | | 1 |
| RISK TOLERANCE | | | | | | | |
| DISCRETIONARY INCOME EXISTS? | | | | | | | |
| INVESTMENTS | | | | | | | |

**Figure 6**

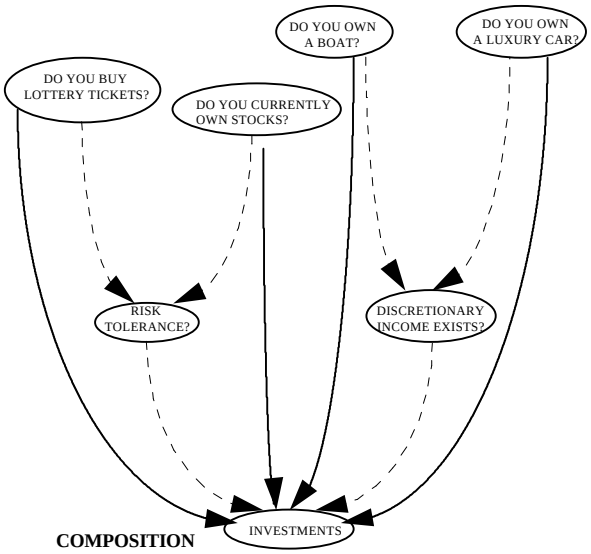



**Figure 2**

VARIABLES IN CONCLUSIONS

| DEPENDENCY RELATIONS OF VARIABLES ON VARIABLES (VARIABLES IN HYPOTHESES) | DO YOU BUY LOTTERY TICKETS? | DO YOU CURRENTLY OWN STOCKS? | DO YOU OWN A BOAT | DO YOU OWN A LUXURY CAR? | RISK TOLERANCE | DISCRETIONARY INCOME EXISTS? | INVESTMENTS |
|---|---|---|---|---|---|---|---|
| DO YOU BUY LOTTERY TICKETS? | | | | | 1 | | 1 |
| DO YOU CURRENTLY OWN STOCKS? | | | | | 1 | | 1 |
| DO YOU OWN A BOAT | | | | | | 1 | 1 |
| DO YOU OWN A LUXURY CAR? | | | | | | 1 | 1 |
| RISK TOLERANCE | | | | | | | 1 |
| DISCRETIONARY INCOME EXISTS? | | | | | | | 1 |
| INVESTMENTS | | | | | | | |

**Figure 8**

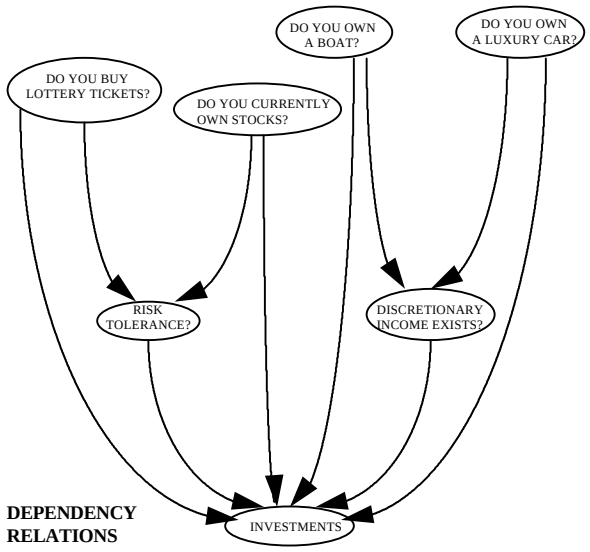



**Figure 9**

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